2023 Maths Olympiads Division J Preparation Kit

MATHS OLYMPIAD

Welcome to the APSMO Maths Olympiads for 2023.

This year, instead of scheduling five competition papers throughout the school year, we are replacing the first competition paper with this Preparation Kit, so that the format of the Maths Olympiads for 2023 will be:

Preparation Kit: Available from February, 2023

Competition 1: Wednesday May 3, 2023

Competition 2: Wednesday June 14, 2023

Competition 3: Wednesday July 26, 2023

Competition 4: Wednesday September 6, 2023

Preparing for the APSMO Maths Olympiads

The purpose of this Preparation Kit is to provide students with an opportunity to familiarise themselves with the concepts, and terminology, that will subsequently be used in the four competition papers for 2023.

For each of the problems in this kit, a number of different solution methods are suggested, so that students can be exposed to multiple ways of approaching mathematical problems.

Examples of how this kit may be used include:

- Reinforcing previously learned concepts and terminology
- Introducing new or different solution methods
- Providing diagrams and animations that support teacher or student explanations
- Offering Follow Up activities for the problems
- Providing opportunity to collaboratively explore student work samples
- Supporting students' own study as a standalone resource

Further questions and solution methods can also be found in the APSMO resource books, available from www.apsmo.edu.au.

2023 Maths Olympiads Division J Preparation Kit

MATHS OLYMPIAD

A. Find the sum of these **20** numbers below.

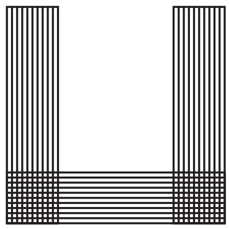
$$321 + 322 + 323 + 324 + 325 +$$
$$321 + 322 + 323 + 324 + 325 +$$
$$321 + 322 + 323 + 324 + 325 +$$
$$321 + 322 + 323 + 324 + 325$$

B. In a bag are **5** red stickers, **4** blue stickers, and **3** green stickers. Emily removes stickers from the bag one at a time, without looking into the bag. She stops when she has three stickers of the same colour.

What is the greatest number of stickers she could take out of the bag?

C. The figure shows three identical **11** cm × **3** cm rectangles that overlap to form two squares.

What is the area of the U-shaped **8**-sided figure?



D. In a game of *Halfsies*, **2** players take turns changing a number. On a player's turn, the player can either subtract **1** from the number or, if the number is even, the player can divide by **2**. A game started with the number **50**. The first player then took his turn.

How many different values are possible after both players have taken two turns each?

E. Emma writes a five-digit number. The thousands and hundreds digits are the same. The hundreds digit is twice the tens digit. The digit in the ten-thousands place is the same as the digit in the ones place. The sum of all five digits is **19**.

There are two possible values for Emma's number. What is the sum of these two numbers?

Write your answers in the boxes on the back.

← ***Keep your answers hidden by folding backwards on this line.***

A.	*Fold here. Keep your answers hidden.*
B.	
C.	
D.	
E.	

Example Solution A

Find the sum of these **20** numbers below.

321 + 322 + 323 + 324 + 325 +
321 + 322 + 323 + 324 + 325 +
321 + 322 + 323 + 324 + 325 +
321 + 322 + 323 + 324 + 325

[Note: The strategies listed below involve Finding A Pattern in the problem.]

Strategy 1: *Multiply each addend in the first row by 4*

Since each term in the first row appears four times, we can multiply each by **4** and add the results together:

(4 × 321) + (4 × 322) + (4 × 323) + (4 × 324) + (4 × 325)

= 1284 + 1288 + 1292 + 1296 + 1300

= **6460**

Strategy 2: *Multiply the sum of the first row by 4*

Since we have four identical rows to add together, we can find the sum of the first row and multiply it by **4**.

The sum of row **1** is: 321 + 322 + 323 + 324 + 325 = 1615.

To get the sum of the numbers in the first row, realise that the average of the five numbers is just the middle number or **323**.

Therefore the sum of the five numbers is **5** × 323 = 1615.

Now that we have the sum of one row, we can now multiply this value by **4** to obtain the sum of all **4** rows: **4** × 1615 = **6460**.

Strategy 3: *Realise that the average of all the numbers is 323*

Since the average of all the numbers is **323** (see strategy **2**), we can just multiply **323** by the number of addends, **20**, to get **323** × **20** = **6460**.

Follow-Up: *Find the sum 909 + 91 + 819 + 181 + 728 + 272 + 637 + 363.* [4000]

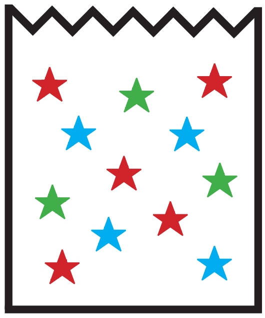
Example Solution B

In a bag are **5** red stickers, **4** blue stickers, and **3** green stickers.
Emily removes stickers from the bag one at a time, without looking into the bag.
She stops when she has three stickers of the same colour.

What is the greatest number of stickers she could take out of the bag?

Strategy: *Use logical thinking*

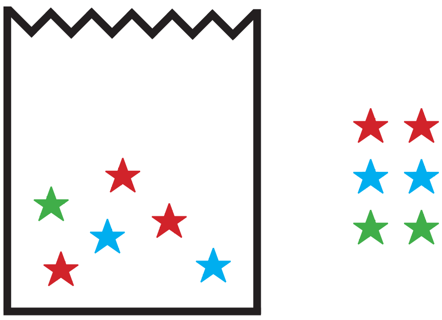
The longest set of stickers that can be removed without getting **3** matching stickers contains **2** reds, **2** blues, and **2** greens.



The next sticker selected must match one of the colours already removed.

Therefore when she selects **7** stickers, she must have at least **3** of the same colour.

[Note: this process is called "examining the worst-case scenario".]

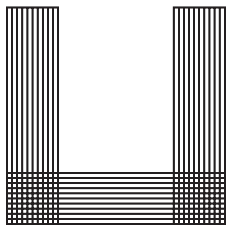


Follow-Up: *Using all of the information in the question, how many stickers must she take in order to guarantee that she will have at least one of each colour?* [**10**]

Example Solution C

The figure shows three identical **11** cm × **3** cm rectangles that overlap to form two squares.

What is the area of the U-shaped **8**-sided figure?



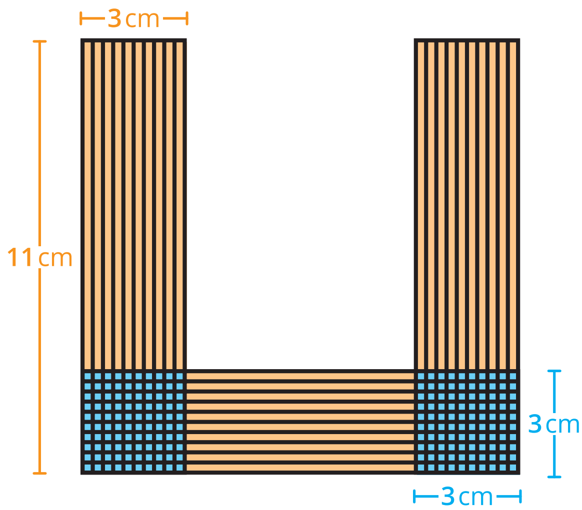
Strategy 1: ***Sum the parts but subtract the overlap***

Consider the left and right vertical rectangles.

Each is **11** cm by **3** cm with an area of **33** cm^2.

There are two of them, so their total area is **66** cm^2.

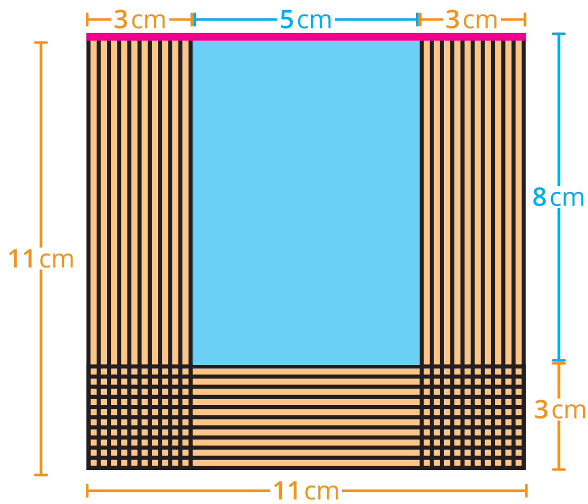
The horizontal rectangle has an area of **33** cm^2 as well, but the two outer corner squares have already been counted, so the area left to be counted is
33 cm^2 - (**2** × **9** cm^2) = **15** cm^2 and
66 cm^2 + **15** cm^2 = **81** cm^2.



Strategy 2: ***Draw a helping line***

Draw a horizontal line from the top left to the top right of the figure, creating a rectangle. The area of the U-shaped **8**-sided figure will be the area of the entire rectangle minus the area of the unshaded rectangle.

The entire rectangle is **11** cm × **11** cm with area **121** cm^2. The area of the unshaded rectangle is **8** cm × **5** cm = **40** cm^2. Therefore, the area of the U-shaped **8**-sided figure is **121** cm^2 – **40** cm^2 = **81** cm^2.



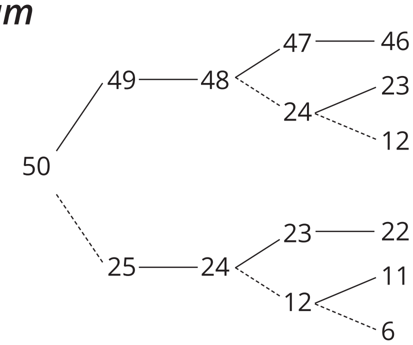
Follow-Up: *Becky made a quilt that was* ***8*** *squares by* ***10*** *squares. The squares on all of the edges were black, forming a border. How many black squares were needed for the border?* [**32**]

Example Solution D

In a game of *Halfsies*, **2** players take turns changing a number.
On a player's turn, the player can either subtract **1** from the number or, if the number is even, the player can divide by **2**.
A game started with the number **50**.
The first player then took his turn.

How many different values are possible after both players ave taken two turns each?

Strategy 1: *List all possible outcomes in a tree diagram*



Therefore there are exactly **6** possible values at the end of two turns.

Strategy 2: *Make a list of all possible outcomes*

Since the number **50** is even, player number one can either divide by **2** or subtract **1**.

Case A: Player one subtracts **1** → **50 – 1** = **49**.
Player two must subtract **1** since **49** is odd → **49 – 1** = **48**.
Player one can either divide by two or subtract one since **48** is even.

Case A1: Player one subtracts **1** → **48 – 1** = **47**.
Player two subtracts **1** → **47 – 1** = **46**.

Case A2: Player one divides by **2** → **48** ÷ **2** = **24**.
Player two can either divide by **2** or subtract **1**.

Case A2a: Player two subtracts **1** → **24 – 1** = **23**.
Case A2b: Player two divides by **2** → **24** ÷ **2** = **12**.

Case B: Player one divides by **2** → **50** ÷ **2** = **25**.
Player two must subtract **1** since **25** is odd → **25 – 1** = **24**.
Player one can either divide by two or subtract one since **24** is even.

Case B1: Player one subtracts **1** → **24 – 1** = **23**.
Player two subtracts **1** → **23 – 1** = **22**.

Case B2: Player one divides by **2** → **24** ÷ **2** = **12**.
Player two can either divide by **2** or subtract **1**.

Case B2a: Player two subtracts **1** → **12 – 1** = **11**.
Case B2b: Player two divides by **2** → **12** ÷ **2** = **6**.

Therefore there are a total of **6** possible values at the end of two turns.

Follow-Up: *Find an initial value to replace* ***50*** *in the above question so that there will be more than* ***6*** *possible outcomes.* [One answer is to start with **32** to get **7** possible final values.]

Example Solution E

Emma writes a five-digit number.
The thousands and hundreds digits are the same.
The hundreds digit is twice the tens digit.
The digit in the ten-thousands place is the same as the digit in the ones place.
The sum of all five digits is **19**.

There are two possible values for Emma's number. What is the sum of these two numbers?

Strategy: ***Use the given information to simplify your search in a targeted way***

If the digits are ***A***, ***B***, ***C***, ***D***, and ***E*** then the number can be represented as ***ABCDE***.

Because the hundreds digit is twice the tens digit, the number is

$$A\ B\ (2D)\ D\ E$$

The thousands digit and the hundreds digits are the same, so the number can now be represented as

$$A\ (2D)\ (2D)\ D\ E$$

The ten-thousands digit and the ones digit are the same, so the number now is

$$A\ (2D)\ (2D)\ D\ A$$

Use the fact that the sum of the five digits is 19.

So $2A + 5D = 19$.

Since ***A*** and ***D*** are single-digit numbers, it is either true that

$$A = 2 \text{ and } D = 3$$

or

$$A = 7 \text{ and } D = 1$$

Therefore the two numbers are: **26 632** and **72 217**.

The sum of the numbers is **98 849**.