APSMO

2023 : DIVISION J

WEDNESDAY 6 SEPTEMBER 2023

OLYMPIAD 4

Total Time Allowed: 30 Minutes

Write your answers in the boxes on the back.

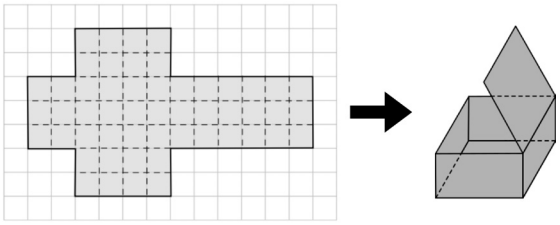
Keep your answers hidden by folding backwards on this line.

4A. The diagram shows a net of a three dimensional rectangular box.

That box is filled with sand.

The sand is then poured out into containers that are 2 units wide by 2 units high by 1 unit deep.

How many of these containers can be filled with the sand from the box?



4B. Every day, Riley adds to his collection of bottle caps by the same amount. In two days from now, he will have 152 bottle caps. Twenty days from now, he will have 242 bottle caps.

What was the number of bottle caps in Riley's collection 7 days ago?

4C. The sum of seven consecutive counting numbers is 2023.

What is the least of these counting numbers??

4D. The lollipops that Scarlett sells at her shop come in packs of 5, 9 or 11.

Scarlett can sell a customer 29 lollipops using 4 packs of 5 lollipops and one pack of 9 lollipops.

She knows any number a customer asks for above 30 lollipops she can make.

What is the largest number of lollipops she is not able to sell to a customer using any combinations of these packs?

4E. The secret passcode for Chloe's Club is a four-digit number.

The passcode has only two prime factors.

One of those primes is a single-digit number.

The sum of the digits of the passcode is 16.

The last three digits of the code are the same. What is the passcode?

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4A.

4B.

4C.

4D.

4E.

Student Name:

Fold here. Keep your answers hidden.

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OLYMPIAD 4

Solutions and Answers

For teacher use only. Not for Distribution.

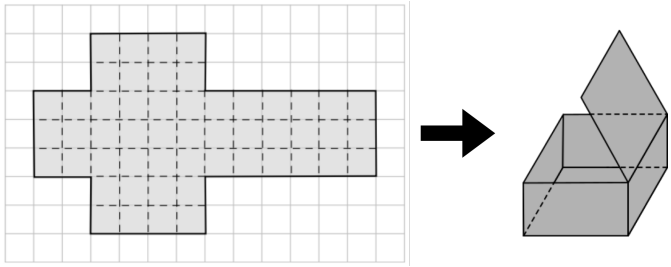
4A: 6	**4B:** 107	**4C:** 286	**4D:** 17	**4E:** 1555

4A. The diagram shows a net of a three dimensional rectangular box.

That box is filled with sand.

The sand is then poured out into containers that are 2 units wide by 2 units high by 1 unit deep.

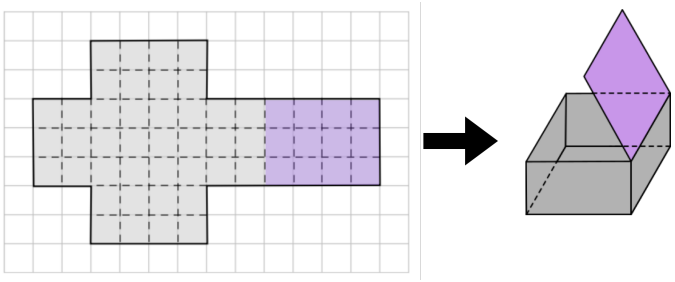
How many of these containers can be filled with the sand from the box?



METHOD 1 *Strategy*: *Compare the dimensions of the box and container to determine their corresponding ratio.*

The dimensions of the box are 4 units wide by 2 units high by 3 units deep.

This can be calculated by recognising that when the four sides are folded, the final 4 units of width and 3 units of depth on the right folds again to become the lid, shown here in purple.

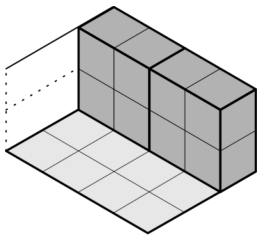


Visualise or sketch the base of the box (4 units wide and 3 units deep) and mark the height as 2 units.

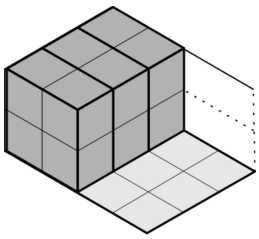
Visualise or sketch the containers that are 2 units wide, 2 units high and 1 unit deep.

While the box and container are the same height, the box is twice as wide and three times as deep.

From here we can calculate that exactly 6 containers can fit inside the box, therefore **6** containers can be filled with the sand from the box.



2 containers wide



3 containers deep

METHOD 2 *Strategy*: *Calculate and divide the volume of the box by the volume of the container.*

To calculate the volume of the box, the first step is to recognise the net in the diagram folds to form a rectangular prism.

This rectangular prism, the box, is 4 units wide by 3 units deep by 2 units high.

4 units x 3 units x 2 units = **24 cubic units**.

The container is 2 units wide by 2 units high by 1 unit deep.

2 units x 2 units x 1 unit = **4 cubic units**.

To find out how many containers can be filled with the sand from the box, we divide the 24 cubic units of the box by the 4 cubic units of a container.

24 cubic units ÷ 4 cubic units = 6.

Therefore the sand from the box will fill **6** containers.

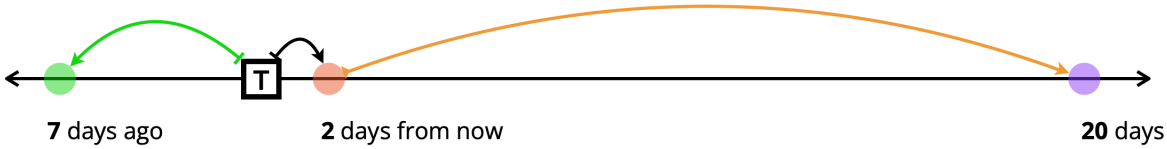
Follow-Up Questions in Members Portal

Video Solutions Uploaded 11/09/23

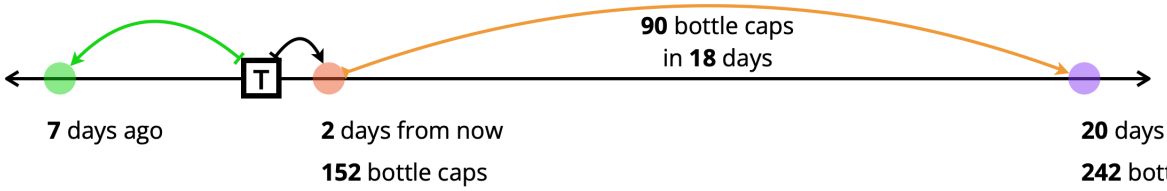
4B. The question is:

Every day, Riley adds to his collection of bottle caps by the same amount. In two days from now, he will have 152 bottle caps. Twenty days from now, he will have 242 bottle caps.
What was the number of bottle caps in Riley's collection 7 days ago?

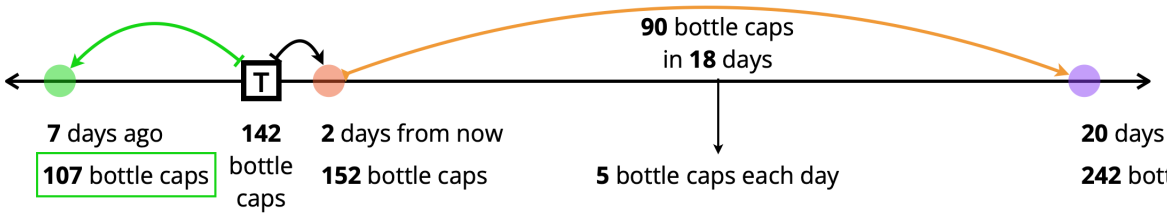
METHOD 1 ***Strategy***: *Use a Timeline.*
Draw a timeline marking today (**T**) and the the other events described in the problem.



Include the information about the number of bottles Riley has two days from now, and 20 days from now, using this to calculate the number of bottles Riley collected in 18 days (90).



Calculate the number of bottle caps Riley collects each day (90 ÷ 18 = 5). We now know Riley has 142 today (152-10) and therefore he had 142 - (7 x 5) seven days ago. Riley had **107** bottle caps in his collection 7 days ago.



Riley had **107** bottle caps in his collection 7 days ago.

METHOD 2 ***Strategy***: *Build a Table.*

Draw a table showing the four days referred to in the problem.

Step 1: Record the number of bottle caps Riley has on days two and 20. Find the difference and divide by the number of days to calculate that Riley collects five bottle caps a day.

Step 2: If Riley has 152 bottle caps two days from now, he therefore has 142 bottle caps today. Record this in the table.

Step 3: To determine how many bottle caps Riley had seven days ago, subtract 7 x 5 from 142. 142 - 35 = **107**

Riley had **107 bottle caps** seven days ago.

	7 days ago	**Today**	2 days from now	20 days from now
Step 1			152	242
Step 2		142	152	242
Step 3	**107**	142	152	242

Step 1: 242 - 152 = 90
90 ÷ 18 = 5 bottle caps a day

Step 2: 152 - (2 x 5) = 142

Step 3: 142 - (7 x 5) = **107**

Follow-Up Questions in Members Portal

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APSMO

2023 : DIVISION J
WEDNESDAY 6 SEPTEMBER 2023

OLYMPIAD 4

4C. The question is:

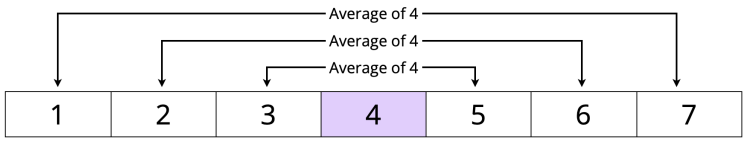
The sum of seven consecutive counting numbers is 2023.
What is the least of these counting numbers?

METHOD 1 ***Strategy*****:** *Simplify the problem.*

The average of an odd number of consecutive numbers is always the middle number of the consecutive numbers.

Let's demonstrate this using the set of consecutive numbers from 1 to 7.

1 + 7, 2 + 6 and 3 + 5 all have an average of 4, the middle number in the set.



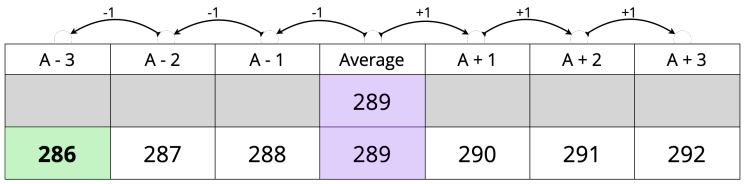
Let's turn back to our problem.

We can calculate that average of the 7 consecutive numbers that have a sum of 2023 is 2023 ÷ 7 = 289.

289 must therefore be the middle of the 7 consecutive numbers.

This number is the fourth number in the sequence, so the smallest number must be three less.

289 − 3 = **286**



METHOD 2 ***Strategy*****:** *Use equations.*

Let the smallest number be represented by **n**.

Then the 7 consecutive numbers are:

n, **n**+1, **n**+2, **n**+3, **n**+4, **n**+5, **n**+6

The sum of these numbers is 2023, so

n + (**n** + 1) + (**n** + 2) + (**n** + 3) + (**n** + 4) + (**n** + 5) + (**n** + 6) = 2023.

The brackets are not needed because the order in which you add numbers does not matter.

n + **n** + **n** + **n** + **n** + **n** + **n** + 1 + 2 + 3 + 4 + 5 + 6 = 2023

(7 × **n**) + 21 = 2023

Because this equation is true, you can subtract 21 from both sides of the equation and the resulting equation is also true.

7 × **n** + 21 = 2023
7 × **n** + 21 − 21 = 2023 − 21.

7 × **n** = 2002.

Division is the opposite of multiplication. 7 × **n** = 2002.

n = 2002 ÷ 7 = 286.

So the smallest of the 7 consecutive numbers is **286**.

Follow-Up Questions in Members Portal

Video Solutions uploaded 11/09/23

4D. The question is:

The lollipops that Scarlett sells at her shop come in packs of 5, 9 or 11.
Scarlett can sell a customer 29 lollipops using 4 packs of 5 lollipops and one pack of 9 lollipops.
She knows any number a customer asks for above 30 lollipops she can make.
What is the largest number of lollipops she is **not** able to sell to a customer using any combinations of these packs?

METHOD 1 ***Strategy***: *Build a Table and Reason Logically #1.*

Scarlett cannot sell a customer 1, 2, 3, or 4 lollipops.

The smallest number of lollipops she can sell is 5.

Build a table that has 5 columns and highlight 5, as well as 9 and 11 to show that she can sell those numbers of lollipops using a single pack.

1	2	3	4	**5**
6	7	8	**9**	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Highlight multiples of 5, 9 and 11 to show that Scarlett can sell customers those amounts of lollipops by combining multiple packs of 5 lollipops, 9 lollipops or 11 lollipops.

1	2	3	4	**5**
6	7	8	**9**	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Scarlett can combine packs of 5, 9 or 11 lollipops in a variety of ways to make different amounts.

Find the multiples of 9 on the table. Any number that appears in a column below a multiple of 9 can be made by adding one or more packs of 5 lollipops.

Find the multiples of 11 on the table. Any number that appears in a column below a multiple of 11 can be made by adding one or more packs of 5 lollipops.

The largest number of lollipops that Scarlett cannot sell combining packs of her lollipops is **17.**

1	2	3	4	**5**
6	7	8	**9**	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

4D. The question is:

The lollipops that Scarlett sells at her shop come in packs of 5, 9 or 11.
Scarlett can sell a customer 29 lollipops using 4 packs of 5 lollipops and one pack of 9 lollipops.
She knows any number a customer asks for above 30 lollipops she can make.
What is the largest number of lollipops she is **not** able to sell to a customer using any combinations of these packs?

METHOD 2 ***Strategy***: *Build a Table and Reason Logically #2.*

Build a table with 11 columns and highlight all multiples of 5, 9 and 11.

Scarlett can make all these amounts for her customers simply by combining multiple packs of 5 lollipops, or 9 lollipops, or 11 lollipops.

1	2	3	4	**5**	6	7	8	**9**	10	**11**
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33

Scarlett can combine different arrangements of lollipop packs to make different amounts. What amounts can she make by combining a pack of 11 lollipops with packs of 5 or 9 lollipops?

Find the multiples of 5 on the table. Any number that appears in a column below a multiple of 5 can be made by adding one or more packs of 11 lollipops.

Find the multiples of 9 on the table. Any number that appears in a column below a multiple of 9 can be made by adding one or more packs of 11 lollipops.

1	2	3	4	**5**	6	7	8	**9**	10	**11**
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33

Look at the remaining amounts that haven't yet been made.

Which can be made by combining packs of 5 and 9 lollipops?

We can make **19** (5+5+9), **23** (9+9=5), **24** (5+5+5+9) and **28** (5+5+9+9).

(These totals are shown with green and purple tiles.)

Any number appearing in a column below these totals can be made by adding one or more packs of 11 lollipops.

1	2	3	4	**5**	6	7	8	**9**	10	**11**
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33

The largest number of lollipops that Scarlett cannot sell combining packs of her lollipops is **17.**

Follow-Up Questions in Members Portal

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APSMO
2023 : DIVISION J
WEDNESDAY 6 SEPTEMBER 2023
MATHS OLYMPIAD
OLYMPIAD 4

4E. The question is:

The secret passcode for Chloe's Club is a four-digit number.
The passcode has only two prime factors.
One of those primes is a single-digit number.
The sum of the digits of the passcode is 16.
The last three digits of the code are the same. What is the passcode?

METHOD 1 ***Strategy*****:** *Use Number Properties to Reduce the Search.*

Let's consider the usefulness of the clues to reduce the size of our search for the passcode and apply them in order.

1) The most helpful clue is that the last three digits are the same. We are only looking for, at most, 2 digits.

We can represent the passcode as **ABBB**.

2) We can continue to strategically reduce our search using the clue that tells us that the sum of the digits in the passcode is **16**.

Let's draw a table to explore ways to total 16 with 4 digits when the last 3 digits are the same.

There is no way for 0, 2, 3, 5, 6, 8 or 9 to total 16 if they are followed by 3 matching digits.

The only digits that meet the criteria are 1, 4 and 7.

A	0	1	2	3	4	5	6	7	8	9
B	x	5	x	x	4	x	x	3	x	x
B	x	5	x	x	4	x	x	3	x	x
B	x	5	x	x	4	x	x	3	x	x
		16			16			16		

3) Finally, let's consider the two remaining clues.

We are told that the passcode has only **2** prime factors and that one of those is a single-digit number.

This eliminates 4444 as a possibility as it is clear to see it can be divided by 2, 4 and 11.

The single digit prime numbers are 2, 3, 5, and 7.

Let's find out if they have 2, 3, 5, or 7 as a factor using division.*

$$2\overline{)1555x} \quad 7\,7\,7\ r1$$

$$3\overline{)1555x} \quad 3\,1\,8\ r1$$

$$5\overline{)1555\checkmark} \quad 3\,1\,1$$

$$7\overline{)1555x} \quad 2\,2\,2\ r1$$

$$2\overline{)7333x} \quad 3\,6\,6\,6\ r1$$

$$3\overline{)7333x} \quad 2\,4\,4\,4\ r1$$

$$5\overline{)7333x} \quad 1\,4\,6\,6\ r3$$

$$7\overline{)7333x} \quad 1\,0\,4\,7\ r4$$

Only 1555 has a single-digit prime. 1555 ÷ 5 = 311

Chloe's passcode is **1555.**

*****Note: Students might also use rules of divisibility such as, 'A number is divisible by 2 if its last digit is even', or 'A number is divisible by 3 if the sum of its digits is divisible by 3', or 'A number is divisible by 5 if the last digit is a 0 or 5' to reduce their search.*

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