APSMO
2021 : DIVISION J
WEDNESDAY 24 MARCH 2021

MATHS OLYMPIAD

OLYMPIAD 1

Total Time Allowed: ***30 Minutes***

Write your answers in the boxes on the back.

← ***Keep your answers hidden by folding backwards on this line.***

1A. Rebecca looks at the clock and sees that it is now 8:00 am.

She says, "Grandma is coming in 243 hours!"

What time will it be when Grandma arrives?

1B. Given the following equations:

□ + ○ = 30

○ + △ = 28

△ + □ = 22

Find the value of: □ + ○ + △

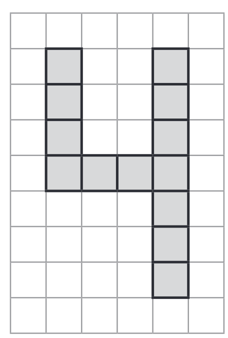
1C. If the pattern continues, what is the letter at the top of the column that contains the number 221?

A	B	C	D	E	F
1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
:	:	:	:	:	19

1D. The shaded shape shown is made up of identical squares.

The perimeter of the shape is 84 cm.

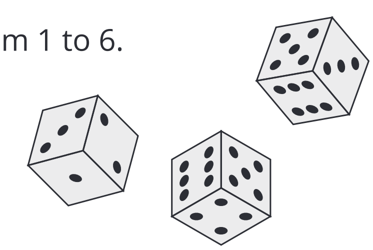
What is the area of the shape, in square centimetres?



1E. A standard die has 6 faces numbered from 1 to 6.

Three standard dice are rolled.

How many different sums are possible for the three numbers on the top face of each die?



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OLYMPIAD

1

Student Name:

1A.

1B.

1C.

1D.

1E.

Fold here. Keep your answers hidden.

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OLYMPIAD 1

MATHS OLYMPIAD

Solutions and Answers

For teacher use only. Not for Distribution.

1A: 11:00 am	1B: 40	1C: *E*	1D: 117 (cm^2)	1E: 16

1A. The question is: What time will it be when Grandma arrives?

METHOD 1 ***Strategy:*** *Build a Table.*

Days later	Hours later	Time of day
1	24	8:00 am
2	48	8:00 am
⋮	⋮	⋮
10	240	8:00 am
	241	9:00 am
	242	10:00 am
	243	11:00 am

1 day, or 24 hours later, it will be **8:00 am** once again.

2 days, or 48 hours later, it will again be **8:00 am**.

After any multiple of 24 hours, it will be **8:00 am**.

Grandma's arrival is 243 hours away.

243 ÷ 24 = 10 remainder 3.

So 10 days later, 10 × 24 = 240 hours will have passed.

There are 243 – 240 = 3 remaining hours until Grandma arrives.

So Grandma will arrive at **8:00 am** + 3 hours = **11:00 am**.

METHOD 2 ***Strategy:*** *Consider the number of hours in a week.*

Hours remaining at 8:00 am
243 – 168 = 75
75 – 24 = 51
51 – 24 = 27
27 – 24 = 3

There are 7 × 24 = 168 hours in a week.

At **8:00 am** exactly one week later, it will be 243 – 168 = 75 hours until Grandma arrives.

One day after that, there will be 75 – 24 = 51 hours left.

A second day after that, there will be 51 – 24 = 27 hours left.

On the third day after that, there will be 27 – 24 = 3 hours left.

It is **8:00 am** when there are 3 hours remaining until Grandma arrives.

So Grandma will arrive at **8:00 am** + 3 hours = **11:00 am**.

Follow-Up: *The bus drops us off at school at 8:45 every morning.*

We board the bus to go home 385 minutes later.

At what time do we start boarding the bus to go home? [3:10 pm]

1B. The question is: Find the value of □ + ○ + △.

Strategy: *Reason logically, and look for relationships.*

METHOD 1: *Combine all three equations.*

We know that □ + ○ = 30, ○ + △ = 28, and △ + □ = 22.

This means that □ + ○ + ○ + △ + △ + □ = 30 + 28 + 22

and so (□ + ○ + △) + (□ + ○ + △) = 80.

Therefore, □ + ○ + △ = 80 ÷ 2 = **40.**

METHOD 2: *Find the difference between two values.*

Since ○ + △ = 28, and △ + □ = 22, ○ must be 28 – 22 = 6 more than □.

We can rewrite this idea as ○ = □ + 6.

□ + ○ = 30, so we now know that □ + □ + 6 = 30.

Therefore, □ + □ = 24 and so □ = 12.

Since □ = 12, and ○ + △ = 28, □ + ○ + △ = 12 + 28 = **40**.

METHOD 3: *Combine two equations, and then subtract the third.*

□ + ○ = 30, and ○ + △ = 28. So □ + ○ + ○ + △ = 30 + 28 = 58.

△ + □ = 22. Subtracting this from □ + ○ + ○ + △, we have ○ + ○ = 58 – 22 = 36.

Therefore, ○ = 36 ÷ 2 = 18, and so △ + □ + ○ = 22 + 18 = **40**.

Follow-Up: *If □ – ○ = 30, and ○ – △ = 28, find the value of □ – △.* [58]

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OLYMPIAD 1

1C. The question is: What is the letter at the top of the column that contains the number 221?

METHOD 1 ***Strategy:*** *Recognise the length of a cycle of numbers.*

We start with **1** in column ***A***, heading to the right.

The next time we are in a similar position, the number is **13**.

This happens after a cycle of **13 – 1 = 12** numbers.

A	*B*	*C*	*D*	*E*	*F*
1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
:	:	:	:	:	19

It may be easier to think of the table as being made up of **12** columns.

If so, there is no need to change direction.

All of the rows now go from left to right.

A	*B*	*C*	*D*	*E*	*F*	*F*	*E*	*D*	*C*	*B*	*A*
1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	…				

We can see that all of the numbers in column ***A*** on the right side are multiples of **12**.

To find a multiple of **12** that is close to **221**:

221 ÷ **12** = **18** remainder **5**.

18 × **12** = **216**.

So **216** is at the end of the **18th** row.

Counting on from **216**, **221** is in column ***E***.

A	*B*	*C*	*D*	*E*	*F*	*F*	*E*	*D*	*C*	*B*	*A*
1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	…				24
											36
											:
										…	216
217	218	219	220	221							

METHOD 2 ***Strategy:*** *Consider patterns in each column.*

In column ***A***, we have **1, 12, 13, 24, 25, 36,** and so on.
(+12 from 1 to 13, from 13 to 25; +12 from 12 to 24, from 24 to 36)

In column ***B***, we have **2, 11, 14, 23, 26, 35,** and so on.
(+12 from 2 to 14, from 14 to 26; +12 from 11 to 23, from 23 to 35)

A	*B*	*C*	*D*	*E*	*F*
1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
24	23	22	21	20	19
25	26	27	28	29	30
36	35	34	33	32	31

In each column, every value is 12 more than the value two rows above.

Since **221** ÷ 12 = **18** remainder **5**, and **5** is in column ***E***, **221** must be in column ***E***.

FOLLOW-UP: *In the 9 × 9 grid at the right, 1 is placed in the central cell (row 5, column 5).*

Consecutive numbers are entered spiralling anticlockwise until 81 is entered in the lower right corner (row 9, column 9).

What is the sum of the three numbers that would be in the other 3 corners?

[195]

			16	15	14	13		
			5	4	3	12		
			6	1	2	11		
			7	8	9	10		
								81

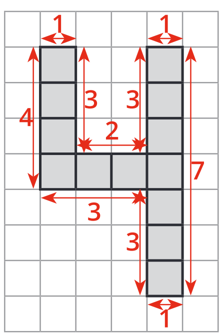
1D. The question is: What is the area of the shape, in square centimetres?

Strategy: *Determine the length of one unit on the diagram.*

METHOD 1: *Count the number of units in the perimeter of the shape.*

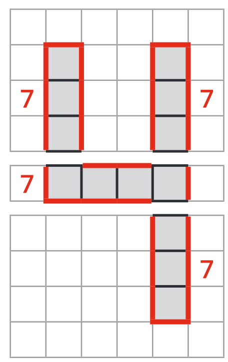
On the diagram, the perimeter is

4 + 3 + 3 + 1 + 7
+ 1 + 3 + 2 + 3 + 1
= 28 units long.



METHOD 2: *Divide the shape into smaller sections.*

The shape in the diagram can be divided into three identical rectangular sections, each with 7 units of perimeter.



The remaining section also contributes 7 units to the perimeter.

So the perimeter is 4 × 7 = 28 units long.

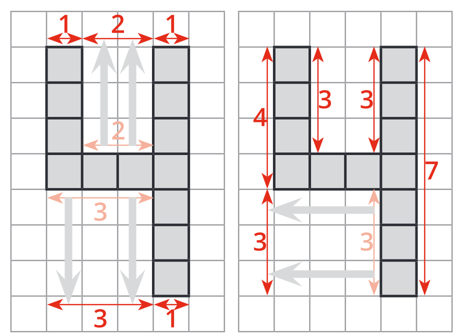
METHOD 3: *Consider horizontal sections and vertical sections separately.*

The perimeter consists of

4 + 4 = 8 horizontal units, and

7 + 3 + 3 + 7 = 20 vertical units.

So the perimeter is 8 + 20 = 28 units long.

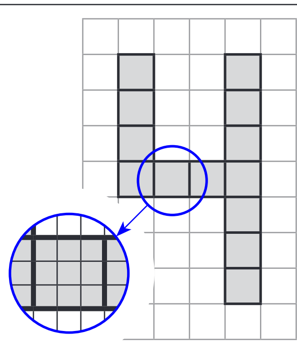


We know that the perimeter of the shape is **84** cm.

On the diagram, the perimeter is 28 units long.

This means that each unit on the diagram is **84** cm ÷ 28 = **3** cm long.

If each unit on the diagram is **3** cm long, then the area of each square is 3 × 3 = 9 cm².



The shape is made up of 13 squares.

Thus, the area of the shape is 13 × 9 cm² = **117** cm².

Follow-Up: *If the area of the shape in the original question had been 325 cm², what would the perimeter of the shape be?* [140 cm]

1E. The question is: How many different sums are possible for the three numbers on the top face of each die?

METHOD 1 *Strategy:* *Create a table.*

Let's consider what happens if we roll two dice.

1st die / 2nd die	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

With two dice, it is possible for the sum to be any whole number value between 2 and 12 inclusive.

If we now roll a third die,

3rd die / 1st & 2nd dice	⚀	⚁	⚂	⚃	⚄	⚅
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12
7	8	9	10	11	12	13
8	9	10	11	12	13	14
9	10	11	12	13	14	15
10	11	12	13	14	15	16
11	12	13	14	15	16	17
12	13	14	15	16	17	18

the sum can be any whole number value between 3 and 18 inclusive.

Counting each value once, we can see that there are **16** different sums.

METHOD 2 *Strategy:* *Consider the possible maximum and minimum sums.*

Each die has the numbers **1** through **6**. ⚀ ⚁ ⚂ ⚃ ⚄ ⚅

Since three dice are rolled,

- the lowest possible sum is ⚀ + ⚀ + ⚀ = 3, and
- the highest possible sum is ⚅ + ⚅ + ⚅ = 18.

Since the numbers on each die are consecutive, all of the numbers from 3 to 18 can be obtained.

So it is possible to have any counting number up to 18, except for the two values **1** and **2**.

Therefore the number of different sums is **18 – 2 = 16**.

Follow-Up: *Suppose each face of a standard die has a different number from the set of the 6 lowest prime numbers.*
If two dice are rolled, how many different sums of the top two numbers are possible? [17; see solution at right]

	2	3	5	7	11	13
2	4	5	7	9	13	15
3	5	6	8	10	14	16
5	7	8	10	12	16	18
7	9	10	12	14	18	20
11	13	14	16	18	22	24
13	15	16	18	20	24	26