APSMO

MATHS GAMES

APSMO

WEDNESDAY 23 MARCH 2022

MATHS GAMES SENIOR 1

Suggested Time: ***30 Minutes***

Write your answers in the boxes on the back.

← ***Keep your answers hidden by folding backwards on this line.***

1A. 24 visitors were collecting audio guides for a museum tour.

18 of the visitors could understand English.

15 of the visitors could understand Mandarin.

2 of the visitors could not understand either English or Mandarin.

How many of the visitors could understand both English and Mandarin?

Hint: How many visitors could not understand English?

1B. A square number results from a whole number being multiplied by itself.

For example, 25 is a square number because 5 × 5 = 25.

How many different square numbers are there, greater than 0 and less than 10 000?

Hint: How many different square numbers are there, greater than 0 and less than 100?

1C. A game can be played by 2 people or by 3 people.

There are 18 games being played, and there are 44 people playing.

How many groups of 3 people are playing?

Hint: How many people would be playing if each game had just 2 players?

1D. Mrs K and Mr D teach in different classrooms.

If Mr D borrows 5 chairs from Mrs K's classroom, they will each have the same number of chairs.

If Mrs K borrows 10 chairs from Mr D's classroom, Mrs K will have four times as many chairs as Mr D.

How many chairs does Mrs K normally have in her classroom?

Hint: You could guess a number of chairs for Mrs K's classroom, and see if it works.

1E. Neil made enough lasagne to fill two same-sized square trays.

One-third of the lasagne has been served from the end of one tray.

Two-fifths of the lasagne has been served from the end of the other tray.

Neil cuts the remaining lasagne in both trays into slices so that all of the slices are the same size, and as large as they can be.

How many slices does he have?

Hint: How might you cut the first lasagne so that you can serve exactly one-third of it?

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SENIOR

1

1A.

Student Name:

1B.

1C.

1D.

1E.

Fold here. Keep your answers hidden.

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MATHS GAMES SENIOR 1

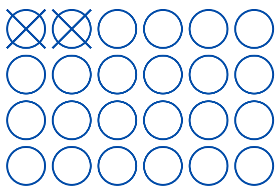
Solutions and Answers

1A: 11	1B: 99	1C: 8	1D: 30	1E: 19

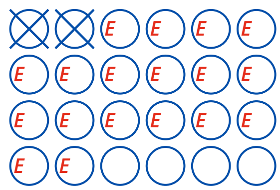
1A. The question is, How many of the visitors could understand both English and Mandarin?

Strategy 1: **Draw a Diagram**

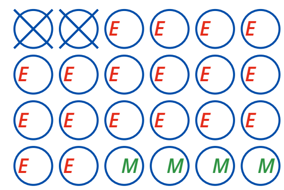
Of the 24 visitors, 2 could not understand either English or Mandarin.



Of the other 24 – 2 = 22 visitors, 18 could understand English.

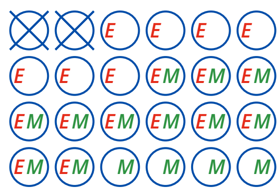


The remaining 22 – 18 = 4 people must all be able to understand Mandarin.



In total, 15 people could understand Mandarin.

The remaining 15 – 4 = 11 Mandarin speakers must also be able to understand English.



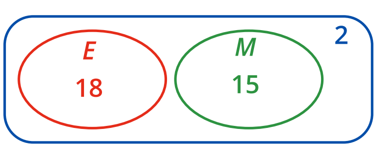
11 of the visitors could understand both English and Mandarin.

Strategy 2: **Draw a Diagram (Alternative Approach)**

There are:

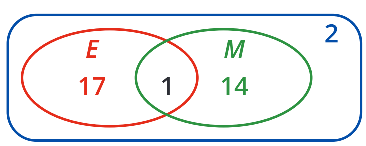
- 18 who understand English,
- 15 who understand Mandarin, and
- 2 who understand neither English nor Mandarin.

If they were all different people, then there would be 18 + 15 + 2 = 35 people.

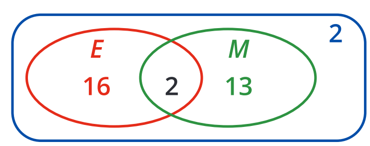


We know that there are only **24** people in total.

If **1** person understood both English and Mandarin, there would be 17 + 14 + 1 + 2 = 34 people.

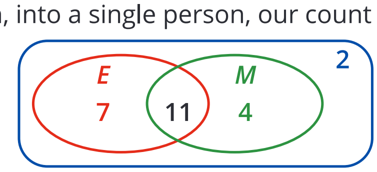


If **2** people understood both English and Mandarin, there would be 16 + 13 + 2 + 2 = 33 people.



We can see that, every time we "combine" two people who understand English and Mandarin, into a single person, our count of people in total will reduce by **1**.

To reduce the total from **35** down to **24**, we will want to do this "combining" **35 – 24 = 11** times.



Therefore, there are **11** people who understand both English and Mandarin.

Follow-Up: *16 of the visitors could understand Spanish. What is the smallest number of people who must be able to understand all three languages - English, Mandarin and Spanish?* [3]

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MATHS GAMES SENIOR 1

1B. The question is, How many different square numbers are there, greater than 0 and less than 10 000?

***Strategy 1:* Guess, Check and Refine, and Draw a Diagram**

We can begin by listing a few numbers, and looking at the magnitudes of their squares.

Number	1	2	5	10								
Square Number	1	4	25	100								

Finding the squares of two-digit numbers is likely to require more effort.

However, we can simplify the process by recognising that, for example:

$$\begin{aligned} 20^2 &= 20 \times 20 \\ &= 2 \times 10 \times 2 \times 10 \\ &= 2 \times 2 \times 10 \times 10 \\ &= 400 \end{aligned}$$

It may be more straightforward if we choose to check the squares of multiples of **10**.

Number	1	2	5	10	20	30	40	50	60	70	80	90
Square Number	1	4	25	100	400	900	1600	2500	3600	4900	6400	8100

So far, all of the squares have been less than 10 000; and at 8100, we are getting much closer.

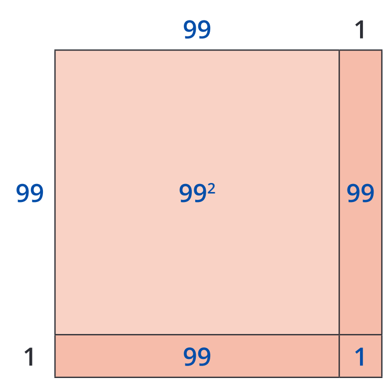
Recognising that 100^2 = 10 000, we can see that 10 000 will be the first square number not to make it into our list.

If we think about square numbers diagrammatically, we can see that 99^2 must be less than 10 000 - we don't actually need to work out that 99^2 = 9801.

Therefore, our list will include all of the numbers from 1^2 to 99^2.

Number	1	2	...	98	99
Square Number	1^2	2^2	...	98^2	99^2

There are **99** square numbers that are greater than **0** and less than **10 000**.



***Strategy 2:* Guess, Check and Refine (Alternative Approach)**

Rather than guessing linearly, we can increase our guess exponentially until we reach, or exceed, 10 000.

Number	1	10	100	
Square Number	1	100	10 000	

We can now deduce that square numbers that are:

- Greater than 0, must be greater than or equal to 1^2 = 1; and
- Less than 10 000, must be strictly less than 100^2 = 10 000.

There must be **99** square numbers that are greater than **0** and less than **10 000**.

Follow-Up: *How many different cubic numbers are there, greater than 0 and less than 8000?* [19]

1C. The question is, How many groups of 3 people are playing?

***Strategy 1:* Guess, Check and Refine**

Let's guess that, of the **18** games, **9** are being played by **3** people, and **18** – **9** = **9** games are being played by **2** people.
In total, there would be **9** × **3** + **9** × **2** = **45** people playing.

3 person games	9			
2 person games	9			
Total people	45			

That's too many. The question says that there are **44** people in total.

Suppose there are **10** games being played by **3** people, and **18** – **10** = **8** games being played by **2** people.
In total, there would be **10** × **3** + **8** × **2** = **46** people playing.

3 person games	9	10		
2 person games	9	8		
Total people	45	46		

That's further from our target than our original guess - Let's guess in the opposite direction.

Suppose there are **8** games being played by **3** people, and **18** – **8** = **10** games being played by **2** people.
In total, there would be **8** × **3** + **10** × **2** = **44** people playing.

3 person games	9	10	8	
2 person games	9	8	10	
Total people	45	46	44	

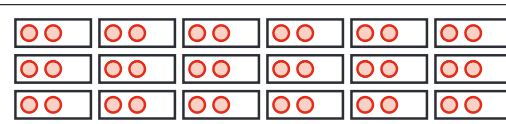
That matches the question.

So there are **8** groups of **3** people playing the game.

***Strategy 2:* Draw a Diagram (1)**

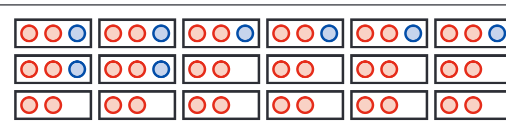
Suppose every one of the **18** games is being played by **2** people.

If so, there would be **18** × **2** = **36** people playing.



Suppose another **44** – **36** = **8** more people turn up to play.

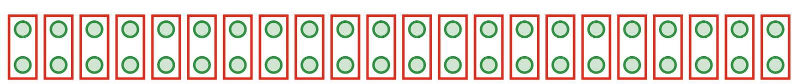
Those people can join **8** of the existing games.



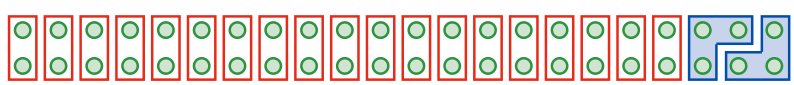
With **44** people playing, there must be **8** groups of **3** people playing the game.

***Strategy 3:* Draw a Diagram (2)**

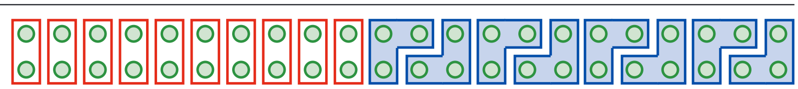
Suppose all **44** people are playing in **22** **2**-person games.



If we redistribute the players from one game to two other games, we end up with **21** games.



We can continue doing this until there are **18** games in total.



With **18** games in total, there must be **8** groups of **3** people.

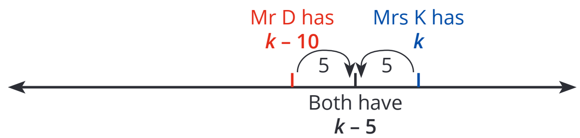
Follow-Up: *How many 3-player games would there be if there were 100 players and 45 games?* [10]

1D. The question is, How many chairs does Mrs K normally have in her classroom?

Strategy: **Draw a Diagram, and Guess, Check and Refine or Use Algebra**

Suppose Mrs K normally has k chairs in her classroom.

If Mr D borrows **5** chairs, then they have the same number of chairs. At this time, Mrs K and Mr D will both have $k - 5$ chairs.



Since Mr D has $k - 5$ chairs after gaining **5** extra chairs, he must normally have $k - 5 - 5 = k - 10$ chairs.

Method 1: Guess, Check and Refine

Suppose Mrs K normally has 20 chairs, and Mr D normally has 20 – 10 = 10 chairs.

After Mrs K borrows **10** chairs from Mr D,

- Mr D will have 10 – 10 = 0 chairs.
- Mrs K will have 20 + 10 = 30 chairs.

Clearly, Mrs K does not now have **4** times as many chairs as Mr D, but let's note this result.

Mrs K normally has	20			
Mr D normally has	10			
After borrowing 10, Mrs K has	30			
After lending 10, Mr D has	0			

We can try different numbers of chairs for Mrs K, to see if we can find a number that works.

Mrs K normally has	20	25	30	
Mr D normally has	10	15	20	
After borrowing 10, Mrs K has	30	35	40	
After lending 10, Mr D has	0	5	10	

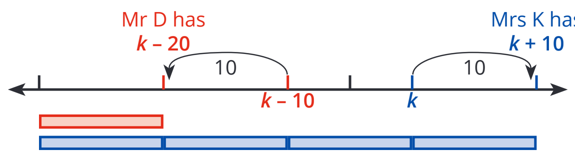
The conditions appear to be satisfied if Mrs K normally has 30 chairs.

Method 2: Use Algebra

If Mrs K borrows **10** chairs from Mr D's classroom, Mrs K will have **4** times as many chairs as Mr D.

When Mrs K borrows **10** chairs from Mr D:

- Mr D will have $k - 10 - 10 = k - 20$ chairs.
- Mrs K will have $k + 10$ chairs.



$k + 10$ is **4** times as many as $k - 20$.

That is:

$$(k - 20) + (k - 20) + (k - 20) + (k - 20) = k + 10$$

$$4k - 80 = k + 10$$

Subtract k from both sides: $3k - 80 = 10$

Add **80** to both sides: $3k = 90$

Divide both sides by **3**: $k = 30$

It looks like Mrs K normally has 30 chairs.

Let's check: Verifying that Mrs K normally has 30 chairs, and Mr D normally has 30 – 10 = 20 chairs.

- After borrowing **5** chairs from Mrs K, Mr D has 20 + 5 = 25 chairs, and Mrs K has 30 – 5 = 25 chairs.
  - They each have the same number of chairs.
- After borrowing **10** chairs from Mr D, Mrs K has 30 + 10 = 40 chairs, and Mr D has 20 – 10 = 10 chairs.
  - Mrs K has **4** times as many chairs as Mr D.

Therefore our working was correct, and Mrs K normally has **30** chairs.

Follow-Up: *Miss H teaches in a classroom down the corridor from Mrs K and Mr D. When they distribute their chairs equally, Miss H gains two chairs. How many chairs does Miss H normally have in her classroom?* [22]

1E. The question is, How many slices does Neil have?

***Strategy 1:* Draw a Diagram (1)**

We could begin by assuming that Neil has cut the lasagne in both trays in the same way.

Since:

- the first tray has had **one-third** of the lasagne removed, and
- the second tray has had **two-fifths** of the lasagne removed,

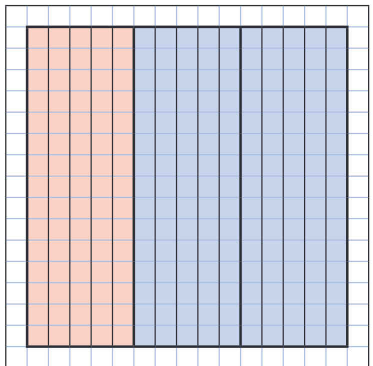
we shall suppose that Neil cut both trays of lasagne into enough equal-sized slices so that he could serve either one-third, or two-fifths.

Since 3 and 5 are relatively prime, the lowest common multiple of 3 and 5 is 3 × 5 = **15**. Therefore if both trays of lasagne were cut in the same way, then the largest possible slice size must be **one-fifteenth** of a lasagne.

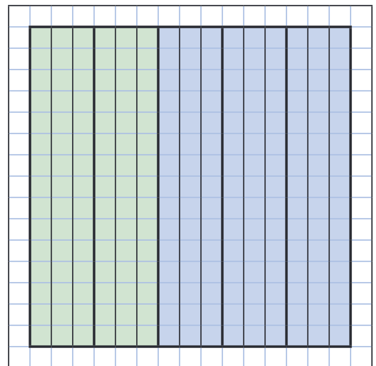
So far, Neil has served:

- $\frac{1}{3} = \frac{5}{15}$, with $\frac{2}{3} = \frac{10}{15}$ remaining, from the first lasagne.
- $\frac{2}{5} = \frac{6}{15}$, with $\frac{3}{5} = \frac{9}{15}$ remaining, from the second lasagne.

In total, $\frac{10}{15} + \frac{9}{15} = \frac{19}{15}$ of a tray remains.



$\frac{1}{3} = \frac{5}{15}$, and $\frac{2}{3} = \frac{10}{15}$



$\frac{2}{5} = \frac{6}{15}$, and $\frac{3}{5} = \frac{9}{15}$

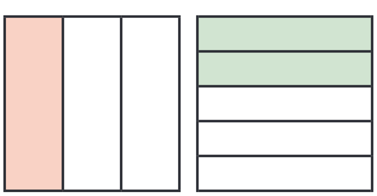
Since the fraction $\frac{19}{15}$ cannot be simplified further, the largest possible slice is $\frac{1}{15}$.

Neil must have **19** slices remaining.

***Strategy 2:* Draw a Diagram (2)**

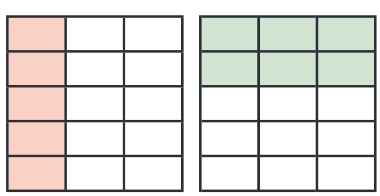
Suppose Neil has cut the two lasagne differently - one into thirds, and one into fifths.

He has served one-third and two-fifths of the lasagne respectively.



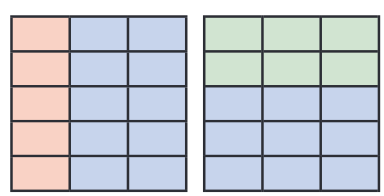
Neil could cut each of the pieces of the first lasagne into fifths, and the pieces of the second lasagne into thirds.

All of the remaining slices would now be the same size.



There are 19 slices remaining.

Since 19 is prime, it is not going to be possible to cut the remaining lasagne into a smaller number of equal-sized slices.



Neil is left with **19** equal-sized slices.

Follow-Up: *Suppose Neil had a third similarly-sized square tray of lasagne, and served five-sixths of it. If he wanted to cut all of the lasagne remaining in all three trays into same-sized pieces, what is the smallest number of pieces he would end up with?* [43]