APSMO

WEDNESDAY 7 SEPTEMBER 2022

MATHS EXPLORER 5

Suggested Time: ***30 Minutes***

5A. Collecting Shells

Jonah, Hugo, and Ava have decided to go to the beach to collect shells.

Jonah collected **15** shells.

Hugo collected the same number of shells as Ava.

When they got home, the three children combined all of the shells they had collected and then shared them out equally between them.

Each person got **11** shells.

How many shells did Ava collect?

5B. Tori's Blocks

Tori has **3** blocks: **1** red, **1** blue, and **1** yellow.

She arranges them in a straight line.

How many different ways can she order them?

Write your answers in the boxes on the back.

←

Keep your answers hidden by folding backwards on this line.

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5A.

5B.

Student Name:

Fold here. Keep your answers hidden.

FOLLOW-ON QUESTIONS

(not compulsory)

To be undertaken by students at the teacher's discretion.
Answers are not to be submitted.

5C. Stacks of Coins

Matilda arranged **18** coins into **3** stacks.

The second stack has **2** more coins than the first stack.

The third stack has **2** more coins than the second stack.

How many coins are there in the second stack?

5D. Isobel's Bangles

Isobel has **3** bangles.

One of them is red, one is green, and one is purple.

Isobel puts all of her bangles on her left arm, without overlapping.

How many different ways can she order them?

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5A. Collecting Shells

Jonah, Hugo, and Ava have decided to go to the beach to collect shells.

Jonah collected **15** shells. Hugo collected the same number of shells as Ava.

When they got home, the three children combined all of the shells they had collected and then shared them out equally between them.

Each person got **11** shells.

How many shells did Ava collect?

Strategy 1: Work Backwards (1)

Jonah, Hugo, and Ava collect shells and then share them out equally.

Let's make a table to talk about the shell situation after each of these events.

After collecting shells	After sharing shells

Here is the information we have been given.

After collecting shells	After sharing shells
Jonah: **15** Hugo: **?** Ava: ***same as Hugo***	

We can think of Jonah, Hugo, and Ava putting together all of the shells they collected, and then sharing them out.

Let's find out how many shells there were in total to share out.

After collecting shells	After sharing shells
Jonah: **15** Hugo: **?** Ava: ***same as Hugo***	Jonah: **11** Hugo: **11** Ava: **11**
	TOTAL: **3** × **11** = **33**

So, now we know that Jonah, Hugo, and Ava collected **33** shells in total.

Jonah collected **15** shells, so Hugo and Ava must have collected **33** – **15** = **18** shells.

If Hugo and Ava together collected **18** shells, and they both collected the same number of shells as each other, then each of them must have collected **18** ÷ **2** = **9** shells.

So Ava must have collected **9** shells.

Let's check: Jonah collected **15** shells, Hugo picked **9**, and Ava picked **9** (because Hugo and Ava collected the same number of shells).

In total they collected **15** + **9** + **9** = **33** shells.

After sharing equally, they will have **33** ÷ **3** = **11** shells each.

This matches the question.

Ava collected **9** shells.

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5A. Collecting Shells

Strategy 2: **Work Backwards (2)**

First, let's go through the same actions as Jonah, Hugo, and Ava.

Ava collects a certain number of shells.	Hugo collects the same number of shells as Ava, so the total number of shells doubles.	Jonah collects 15 shells, which gets added to the total.	Jonah, Hugo, and Ava share the shells equally among the 3 of them.	They each got 11 shells.
? shells	multiply by 2 → ? shells	add 15 → ? shells	divide by 3 →	11 shells

Now, we can work backwards to find the total number of shells collected by Ava.

9 shells	divide by 2 ←	18 shells	subtract 15 ←	33 shells	multiply by 3 ←	11 shells
Ava collected 9 shells.	Hugo and Ava collected the same number of shells, so we halve their total.	Jonah's 15 shells can be subtracted from the total to find the number of shells collected by Hugo and Ava.	Before equally sharing the shells, the total number they collected must have been 3 times the number of shells they each ended up with.	They each got 11 shells.

Let's check: Ava collected 9 shells.
Hugo collected the same number of shells as Ava, so he also collected 9 shells.
Jonah collected 15 shells.
So the total number of shells collected was $9 + 9 + 15 = 33$.
Dividing all the shells equally between the three friends meant that they each got $33 \div 3 = 11$ shells.
This matches the question.
Ava collected **9** shells.

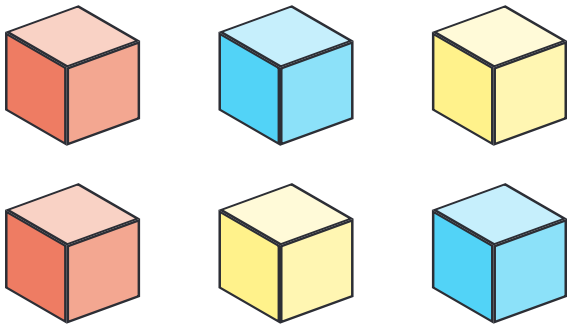
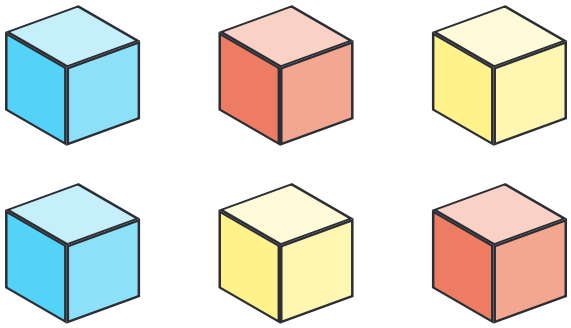
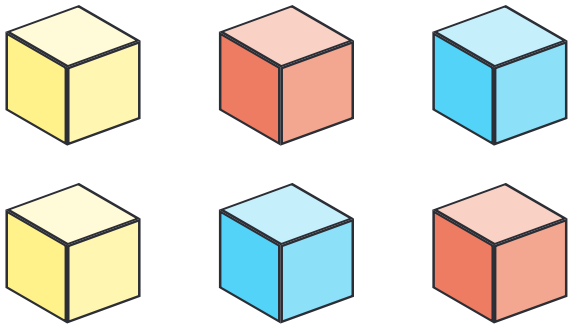
5B. Tori's Blocks

Tori has **3** blocks: **1** red, **1** blue, and **1** yellow.

She arranges them in a straight line.

How many different ways can she order them?

Strategy 1: **Make an Organised List by Using Concrete Materials**

Let's place the red block first. We can have red, blue, yellow. We can have red, yellow, blue. This gives Tori **2** different possibilities when she places the red block first.	
Now, let's place the blue block first. We can have blue, red, yellow. We can have blue, yellow, red. This gives Tori **2** different possibilities when she places the blue block first.	
Finally, let's place the yellow block first. We can have yellow, red, blue. We can have yellow, blue, red. This gives Tori 2 different possibilities when she places the yellow block first.	

There are **3** different colours of blocks, and **2** different ways to arrange the blocks for each colour that is placed first, so there are **3** × **2** = **6** different combinations.

Altogether, there are **6** different ways Tori can arrange her blocks.

5B. Tori's Blocks

Strategy 2: Make an Organised List

We can also make an organised list in the following way.

Let's say Tori places the red block first.

How many different ways can Tori order them if she places the red block first?

She's only got two more blocks to place. So she can put the blue one next, or the yellow one next. The third block is whatever is left.

Block 1	Block 2	Block 3
Red	Blue	Yellow
Red	Yellow	Blue

What if Tori places the blue block first?

Block 1	Block 2	Block 3
Red	Blue	Yellow
Red	Yellow	Blue
Blue	Red	Yellow
Blue	Yellow	Red

Can you see a pattern?

What if Tori places the yellow block first?

Block 1	Block 2	Block 3
Red	Blue	Yellow
Red	Yellow	Blue
Blue	Red	Yellow
Blue	Yellow	Red
Yellow	Red	Blue
Yellow	Blue	Red

That's all of them. Let's count them up. Tori can order her blocks in **6** different ways.

Strategy 3: Draw a Diagram

We can draw a diagram where each group represents a different combination.

Let's have Tori place the red block first.

She can then place either the blue or yellow block next.

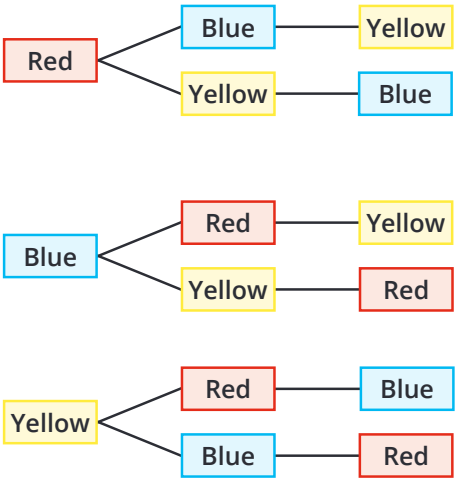
If she places the blue block after the red block, she then only has to place the yellow block.

If she places the yellow block after the red block, she then only has to place the blue block.

Tori can then repeat this process by placing the blue block first, followed by the red block then the yellow block.

She can then change the order and place the yellow, then the red block after the blue block.

Finally, Tori can place the yellow block followed by the red block then the blue block. She can also change the order and place the blue block and the red block after the yellow block.



Tori can order her blocks in **6** different combinations.

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5C. Stacks of Coins (FOLLOW-ON QUESTION)

Matilda arranged **18** coins into **3** stacks.

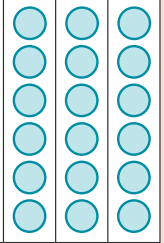
The second stack has **2** more coins than the first stack.

The third stack has **2** more coins than the second stack.

How many coins are there in the second stack?

Strategy 1: Work Backwards

Let's begin by distributing the **18** coins evenly into the three stacks.

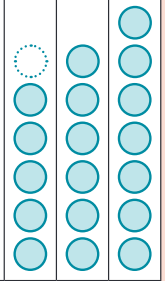


Stack Number	1	2	3

There are 6 coins on each stack.

The first stack is supposed to be the shortest.

Let's take a coin off Stack 1 and give it to Stack 3, which is meant to be the tallest.



Stack Number	1	2	3

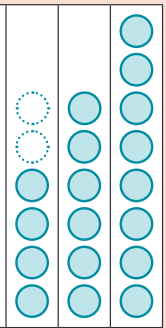
With this arrangement,

- Stack 2 has one coin more than Stack 1.
- Stack 3 has one coin more than Stack 2.

We want Stack 2 to have two coins more than Stack 1, and Stack 3 to have two coins more than Stack 2.

Let's repeat the process.

We'll take one more coin off Stack 1, and give it to Stack 3.



Stack Number	1	2	3

With this arrangement,

- Stack 2 has two coins more than Stack 1.
- Stack 3 has two coins more than Stack 2.

That matches the question.

We can see that there are **6** coins in the second stack.

Strategy 2: Build a Table and Find a Pattern

	1st Stack	2nd Stack	3rd Stack	Total Coins
Let's start with just 1 coin in the first stack. There would then be 1 + 2 = 3 coins in the second stack, and 3 + 2 = 5 coins in the third stack, for a total of 1 + 3 + 5 = 9 coins.	1	3	5	1 + 3 + 5 = 9
If there were 2 coins in the first stack, there would be 4 in the second and 6 in the third, for a total of 2 + 4 + 6 = 12 coins.	2	4	6	2 + 4 + 6 = 12
Each time we increase the number of coins in the first stack by 1, the total number of coins increases by 3. Why does this happen?	3	5	7	3 + 5 + 7 = 15
Following the pattern, we can see that having 4 coins in the first stack would result in a total of 4 + 6 + 8 = 18 coins. This matches the question.	4	6	8	4 + 6 + 8 = 18

There are **6** coins in the second stack.

5D. Isobel's Bangles (FOLLOW-ON QUESTION)

Isobel has **3** bangles.

One of them is red, one is green, and one is purple.

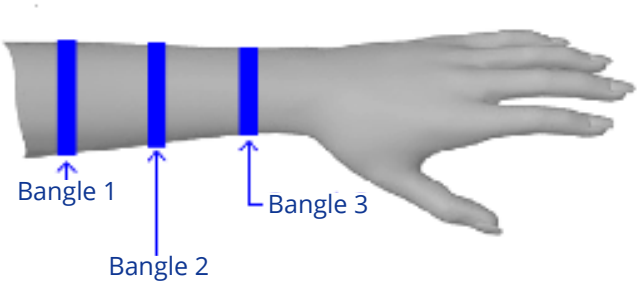
Isobel puts all of her bangles on her left arm, without overlapping.

How many different ways can she order them?

Strategy: **Make an Organised List**

Let's think for a moment about how Isobel's arm looks.

- When she puts on the first bangle, it will be the one closest to her elbow.
- When she puts on the third bangle, it will be one closest to her hand.
- The second bangle will sit in between.



Let's say Isobel puts on the red bangle first.

How many ways can she make them look if the red one went on first?

She's only got two more to put on. So she can put on the green one next, or the purple one next. The third bangle is whatever is left.

Bangle 1	Bangle 2	Bangle 3
Red	Green	Purple
Red	Purple	Green

What if Isobel puts on the green bangle first?

Bangle 1	Bangle 2	Bangle 3
Red	Green	Purple
Red	Purple	Green
Green	Red	Purple
Green	Purple	Red

Can you see a pattern?

What if Isobel puts on the purple bangle first?

Bangle 1	Bangle 2	Bangle 3
Red	Green	Purple
Red	Purple	Green
Green	Red	Purple
Green	Purple	Red
Purple	Red	Green
Purple	Green	Red

That's all of them. Let's count them up.

Isobel can order her bangles in **6** different ways.