About This Book

When I was studying to become a secondary school mathematics teacher, my methods lecturer (the wonderful Anne Prescott) presented the class with the following problem:

> In a stationery store, pencils have one price and pens have another price. Two pencils and three pens cost 78c. But three pencils and two pens cost 72c. How much does one pencil cost?

The problem was accompanied with the caveat that we were not allowed to use algebra to solve it. I have a vivid memory of the difficulty I had in working out a solution that used no algebra. I cannot recall if I was successful at the time.

A few years later, I gave the same problem to a child. He thought about it for a little while and then informed me that it was a pattern. The conversation went something like this:

> Me: Can you show me the pattern?
>
> Child: Well, so you have two pencils and three pens. You go to three pencils and two pens. So you can probably go to four pencils and one pen.

His insight, that there was a pattern that corresponded to the difference in price, has informed some of the methods that are shown in this book. Others, from students to academic and teaching colleagues, have contributed equally interesting methods and ideas. My role in all of this has been to collate and consider all of the methods that I have seen, and consolidate them in a way that might be helpful for teaching.

The resources in this book are designed to support the teaching and learning of mathematical problem solving, particularly for students for whom such a pursuit is relatively new. To this end, the book is structured with two particular parameters in mind.

1. Problems are Sequenced by Difficulty

Set 1 problems have been chosen to be less cognitively demanding than those in Set 15. The teacher of mathematics "challenges the curiosity of his students by setting them problems proportionate to their knowledge"[1], and teachers are best placed to select problems that suit the Zones of Proximal Development for their students.

Notably, teaching through problem solving also supports the teaching of mixed-ability classes. A large-scale study described by Boaler and Staples[2] indicated that students in heterogenous groups, taught using a problem-solving approach, improved more quickly than ability-grouped students taught using a more traditional approach. After two years, students in the mixed-ability setting were significantly outperforming their traditionally taught counterparts, with the greatest achievement advantage shown by students who, at the beginning of the study, had been assessed to already be relatively high achieving. Students appeared to be developing deeper understanding from the act of explaining work to others.

2. Problems are Mixed by Strategy

There are ten problems in each set, and the answers to each problem in the set are presented with different solution strategies.

Mixing problems has two significant consequences for learning. Firstly, students must learn to choose an appropriate strategy to solve the problem; and secondly, problems that are structurally similar are, as a result, experienced at spaced intervals.

The literature indicates that both of these characteristics are beneficial. For example, Rohrer, Dedrick and Stershic describe a study where students were assigned the same set of practice

1. Polya, G. (1945). *How to solve it*. Princeton University Press.

2. Boaler, J. & Staples, M. (2008). Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School. *Teachers College Record, 110*(3), pp. 608–645.

problems over the same length of time[3]. Some of the students had the problems arranged in blocks requiring the same solution strategy, while others had the problems interleaved. The results of both an immediate and a delayed test demonstrated that students who engaged with the mixed practice gained higher test scores.

Interestingly, an earlier study[4] also noted that the superior results for the mixed-practice students were only evident in their tests. Scores for the practice sessions favoured the students who practised with problems arranged in blocks. This suggests that it is worth taking a long view when considering how we assign the mathematics practice, as an immediate reduction in practice scores is not necessarily indicative of an inappropriate strategy for learning.

What is Problem Solving?

Problem solving is naturally characterised by some kind of struggle. Lenchner contrasts problem solving with mathematical exercises, observing that

> An exercise is a task for which a procedure for solving is already known.
>
> A problem is more complex because the strategy for solving may not be immediately apparent; solving a problem requires some degree of creativity or originality on the part of the problem solver.[5]

As teachers of mathematics, we require a complex and nuanced set of skills. We obviously need to know the mathematics, and we need to understand how students learn. We also need to have a good feel for how, and when, and for how long, to let the students struggle, and how to interpret the results of the struggle when they appear.

Giving students time to struggle, perhaps unproductively, may be counter-intuitive, but it has been shown to be a good investment in their mathematical development[6]. It is, however, potentially confronting for the teacher who is tasked with interpreting what the student has produced, and deciding whether or not it has validity. As Thurston notes,

> Young children come up with many ingenious devices to work out mathematical questions, but teachers usually discourage any nonconventional approach - partly because it is not easy to understand what a child is thinking or trying to say, and partly because the teachers think it's not okay to use an alternative method or explanation.[7]

If this book had a single aim, then it would be to address this particular point. The purpose is to support teachers to make sense of different strategies that students may use, as they learn to solve mathematical problems. It is not intended to demonstrate how to teach, so much as to assist with the problem of interpretation.

When students have been given time to think about a problem, and encouraged to solve it in any way they like, they may come up with any number of ingenious methods. It is my hope that the solution pages will provide enough structure to be able to decipher what it is the student is trying to convey, and perhaps offer ideas for how to help the student to demonstrate their method coherently to the rest of the class.

The sharing of a solution strategy is, I think, the most potent part of the learning experience. Firstly, it benefits the student who is doing the explaining, because they need to articulate what it is that they have done; they must understand it before they can explain it to someone else[8].

It also benefits the rest of the class as it exposes everyone to a credible, achievable method, because it is a method that makes sense to a peer. From this, students may gain increased confidence in their own ability to address unfamiliar problems. Even when students have solved a problem to their own satisfaction, exposure to other methods is fundamental to their development as effective and sophisticated problem solvers, with each alternative strategy contributing to the "scaffolding, with many interconnected supports"[7] that allows us to build powerful conceptual structures.

3. Rohrer, D., Dedrick, R.F. & Stershic, S. (2015). Interleaved practice improves mathematics learning. *Journal of Educational Psychology, 17*(3), pp. 900-908.

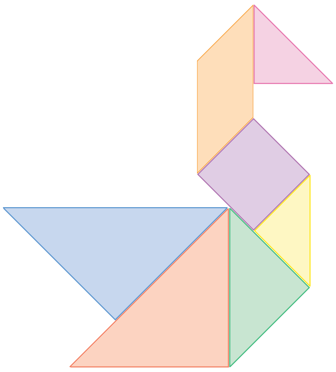
4. Rohrer, D. & Taylor, K. (2007). The shuffling of mathematics problems improves learning. *Instructional Science, 35*, pp. 481-498.

5. Lenchner, G. (2005). *Creative problem solving in school mathematics, 2nd ed*.

6. Kapur, M. (2010). Productive failure in mathematical problem solving. *Instructional Science, 38*, pp. 523–550.

7. Thurston, W.P. (1990). Mathematical education. *Notices of the AMS, 37*, pp. 844-850.

8. Goos, M., Vale, C. & Stillman, G. (2016). *Teaching secondary school mathematics, 2nd ed*. Routledge.



The Strategies

Mathematics is a creative subject requiring abstract thought. Children naturally reason and use creative strategies when they seek patterns and relationships that will enable them to solve challenging unfamiliar problems. The generalisations they make can then be used to solve problems with the same mathematical structure.

Through the process of problem solving and class discussion of the strategies used, children will also develop skills they can use when faced with more unfamiliar problems, so by Years 5-6 they will be able to:

- Describe and represent mathematical situations in a variety of ways
- Select and apply appropriate problem-solving strategies in undertaking investigations
- Give valid reasons for supporting one possible solution over another.

Problems can often be solved in many different ways. For this reason, different methods of solution will be suggested for each problem, with particular emphasis on:

Guess, Check and Refine

With this strategy, the student makes a reasonable guess of the answer, and then checks the guess against the conditions of the problem. If the first guess is not correct, the student obtains more information that may lead to the answer.

Beginners in particular are urged to use "Guess, Check and Refine" often, until they catch the "feel" of solving problems.

Draw a Diagram

If a problem is not illustrated, sometimes it is helpful for the student to draw a diagram.

A picture may reveal information that may not be obvious just by reading the problem.

It is also useful for keeping track of where the student is up to in a multi-step problem.

Find a Pattern

One of the most frequently used problem solving strategies is that of recognising and extending a pattern.

Students can often simplify a difficult problem by identifying a pattern in it, and then applying that pattern to the problem situation.

Build a Table

A table displays information so that it is easily located and understood, and missing information becomes obvious.

If students are not given the data for a problem, and must generate it themselves, a table is an excellent way to record what they have done so they don't have to repeat their efforts.

A table can also be invaluable for detecting significant patterns.

Work Backwards

If a problem describes a procedure and then specifies the final result, this method usually makes the problem much easier to solve.

Make an Organised List

Listing every possibility in an organised way is an important tool.

How students organise the data often reveals additional information.

Solve a Simpler Related Problem

Many hard problems are actually relatively straightforward problems that have been extended to larger numbers.

Replacing the large numbers with smaller numbers can introduce patterns that allow insights into how to solve the original problem.

Eliminate All But One Possibility

Deciding what a quantity is not, can narrow the field to a very few possibilities.

These can then be tested against the conditions of the original problem.

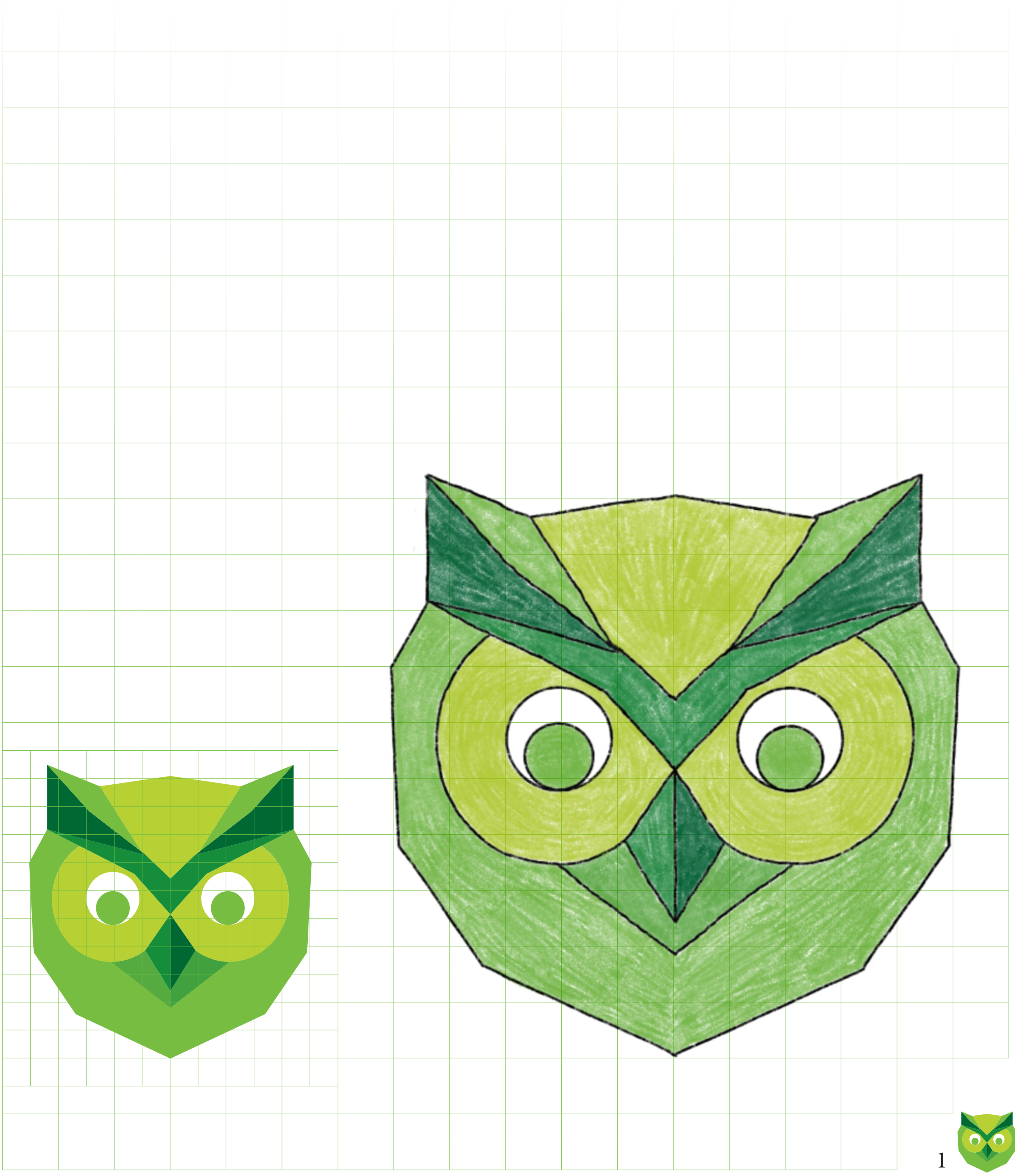
Convert to a More Convenient Form

There are times when changing some of the conditions of a problem makes a solution clearer or more convenient.

Divide a Complex Shape

Sometimes it is possible to divide an unusual shape into two or more common shapes that are easier to work with.

Problem Set 1



Problem Set 1

1 Millie has twenty pens and pencils in her pencil case.

She has four more pencils than pens.

How many pens does she have?

2 There are 12 people in a boat.

There are 8 more men than women in the boat.

How many women are in the boat?

3 Billy is writing a sequence of numbers.

He starts with 1, then adds 2, 3, 4, 1, 2, 3, 4, 1 ... and so on, repeatedly adding 2, then 3, then 4, then 1, like this:

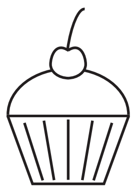
Billy's sequence:	1,	3,	6,	10,	11,	...
	+ 2	+ 3	+ 4	+ 1	+ 2	+ 3

What is the 20th number in Billy's sequence?

4 Charlie wants to celebrate his birthday by bringing cupcakes to school for everyone in Year 6.

He can buy cupcakes in packets of 12.

If there are 53 students and two teachers, how many packets of cupcakes will Charlie need to buy for everyone to each have one cupcake?



5 Miss Humphrey bought a packet of stickers to give as prizes to her class.

She gave half of them to Mrs Sutton to use for the class next door.

At the end of the week, she handed out one sticker each to Rhys, Shen and Taylor.

She has twelve stickers left.

How many stickers did Miss Humphrey have before giving any of them away?

6 Sandy writes every whole number from 1 to 100 without skipping any numbers.
How many times will Sandy write the digit “2”?

7 What is the value of the following?

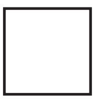
123 + 123 + 123 + 123 + 123 +
123 + 123 + 123 + 123 + 123 +
123 + 123 + 123 + 123 + 123 +
123 + 123 + 123 + 123 + 123

8 There is a three-digit number on my car’s number plate.
The hundreds digit is one more than the tens digit.
The ones digit is equal to the hundreds digit multiplied by the tens digit.
No two digits are the same.
What is the three-digit number?

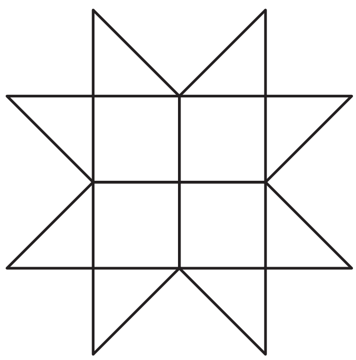
9 What is the value of the following?

268 + 1375 + 6179 – 168 – 1275 – 6079

10 I am using square tiles to make a pattern.
The tiles can be cut to make different shapes.

All of the tiles are this size: 

What is the smallest number of tiles I will need to make this 8-pointed star?



Millie's Pencil Case

Problem: Millie has twenty pens and pencils in her pencil case.
She has four more pencils than pens.
How many pens does she have?

Strategy: Guess, Check and Refine

Let's guess that there are 10 pens in her pencil case.
If so, there would be 10 + 4 = 14 pencils.
In total, there would be 10 + 14 = **24** pens and pencils.

Pens:	10			
Pencils:	14			
Total:	24			

That's too many. There should be **20** pens and pencils all together.

If there were 9 pens, there would be 9 + 4 = 13 pencils.
In total, there would be 9 + 13 = **22** pens and pencils.

Pens:	10	9		
Pencils:	14	13		
Total:	24	22		

Let's try taking away 1 more pen, for a total of 8 pens.
If so, there would be 8 + 4 = 12 pencils.
In total, there would be 8 + 12 = **20** pens and pencils all together.

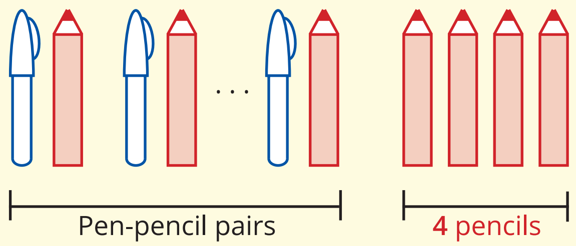
Pens:	10	9	8	
Pencils:	14	13	12	
Total:	24	22	20	

That matches the question, so there are **8** pens in Millie's pencil case.

Strategy: Draw a Diagram, and Reason Logically

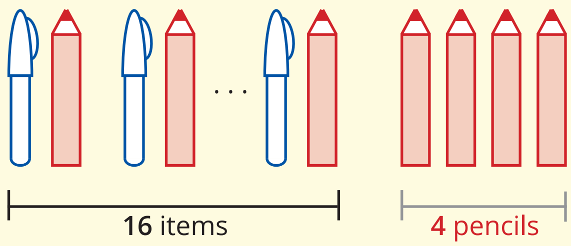
Suppose we paired up each pen with a pencil.

With 4 more pencils than pens, there will be 4 unpaired pencils.



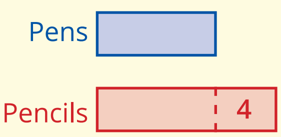
There will be **20** – 4 = **16** items in pairs.

Therefore there are **16** ÷ **2** = **8** pens.

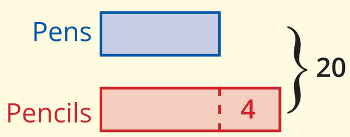


Strategy: Draw a Diagram

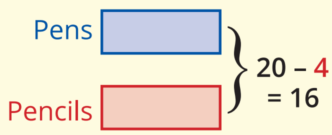
There are 4 more pencils than pens.



There are **20** pens and pencils in total.



We can take away 4 pencils and adjust the total.



Now, there are the same number of pens as pencils.

Therefore there are **16** ÷ **2** = **8** pens.

Problem: There are 12 people in a boat.
There are 8 more men than women in the boat.
How many women are in the boat?

Strategy: Draw a Diagram

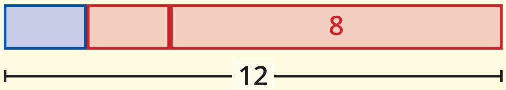
Let's use a bar to represent the number of women.

Number of Women: 

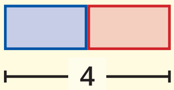
There are eight more men than women.

Number of Men: 

There are **12** people in total.

Total People: 

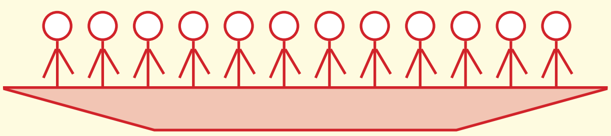
From the diagram, we can see that these two bars together would comprise **12** – **8** = **4** people.



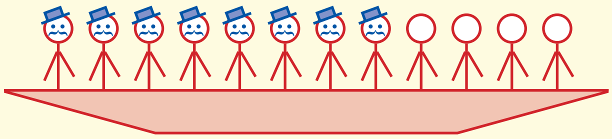
The number of women must be **4** ÷ **2** = **2**.

Strategy: Draw a Diagram (Alternative Approach)

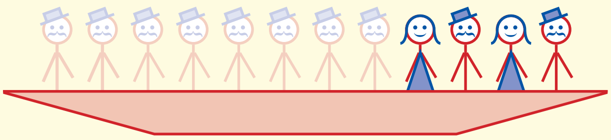
We can draw the boat with **12** people in it.



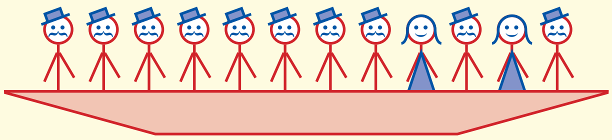
Let's make eight of the people men.



If we took those eight men away, there must be just as many men as there were women.



So there must be **2** women in the boat.



Strategy: Guess, Check and Refine

Let's guess that there are **4** women.

With **8** more men than women, that's **4** + **8** = **12** men.

4 women + **12** men = **16** people.

We want the boat to have **12** people.

If there are **3** women, there would be **3** + **8** = **11** men.

3 women + **11** men = **14** people.

If there are **2** women, there would be **2** + **8** = **10** men.

2 women + **10** men = **12** people.

That matches the question.

There must be **2** women on the boat.

Billy's Sequence

Problem:

Billy is writing a sequence of numbers.

He starts with 1, then adds 2, 3, 4, 1, 2, 3, 4, 1 ... and so on, repeatedly adding 2, then 3, then 4, then 1, like this:

Billy's sequence:	1,	3,	6,	10,	11,	...
	+ 2	+ 3	+ 4	+ 1	+ 2	+ 3

What is the 20th number in Billy's sequence?

Strategy: Find a Pattern

Billy's sequence looks like this.

Billy's Sequence:	1,	3,	6,	10,	11,	...
	+ 2	+ 3	+ 4	+ 1	+ 2	+ 3

To find the **20**th number, we can just keep adding numbers according to Billy's pattern.

1,	3,	6,	10,	11,	13,	16,	20,	21,	23,	26,	30,	31,	33,	36,	40,	41,	43,	46,	50
+ 2	+ 3	+ 4	+ 1	+ 2	+ 3	+ 4	+ 1	+ 2	+ 3	+ 4	+ 1	+ 2	+ 3	+ 4	+ 1	+ 2	+ 3	+ 4	

The 20th number in Billy's sequence is **50**.

Strategy: Find a Pattern (Alternative Approach)

Billy is creating his sequence by starting with 1 and then adding 2, then 3, then 4, then 1 ... and so on.

So the **20**th number in the sequence is the result of the following sum:

$$1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4$$

Method 1: Consider groups of (1 + 2 + 3 + 4).

Suppose we wrote the sum as:

$$1 + 2 + 3 + 4 \quad + \ 1 + 2 + 3 + 4 \quad + \ 1 + 2 + 3 + 4 \quad + \ 1 + 2 + 3 + 4 \quad + \ 1 + 2 + 3 + 4$$

Since **1 + 2 + 3 + 4 = 10**, we can see that the sum is **10 + 10 + 10 + 10 + 10 = 50**.

Method 2: Rearrange to be a series of multiplications.

We could write the sum as:

$$1 + 1 + 1 + 1 + 1 \quad + 2 + 2 + 2 + 2 + 2 \quad + 3 + 3 + 3 + 3 + 3 \quad + 4 + 4 + 4 + 4 + 4$$

The sum is equal to $5 \times 1 + 5 \times 2 + 5 \times 3 + 5 \times 4 = 5 + 10 + 15 + 20 =$ **50**.

Method 3: Find "Friends of Five".

We can group the values two at a time to make "friends of five":

$$1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4$$

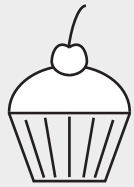
The sum is equal to **10 × 5 = 50**.

Charlie's Cupcakes

Problem: Charlie wants to celebrate his birthday by bringing cupcakes to school for everyone in Year 6.

He can buy cupcakes in packets of 12.

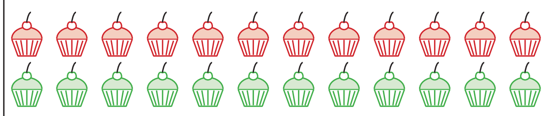
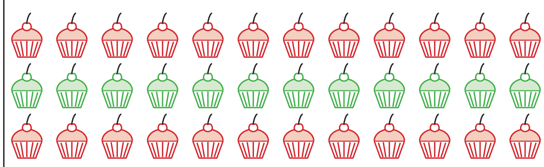
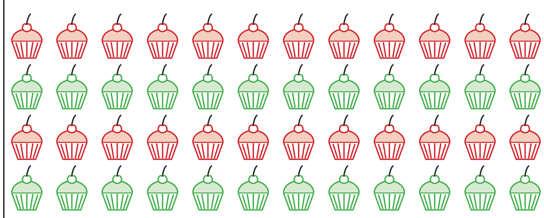
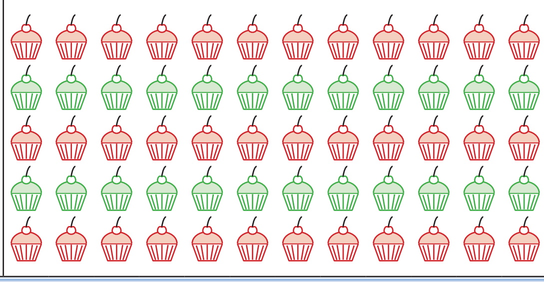
If there are 53 students and two teachers, how many packets of cupcakes will Charlie need to buy for everyone to each have one cupcake?



Strategy: Draw a Diagram, and Build a Table

There are 53 + 2 = 55 people, so Charlie needs at least 55 cupcakes.

Let's build a table to show how many cupcakes Charlie would need to buy.

No. of Packets	Cupcakes	No. of Cupcakes
1		12
2		24
3		36
4		48
5		60

With 4 packets, there still aren't enough cupcakes.

5 packets will have enough cupcakes for 55 people.

So Charlie needs to buy 5 packets of cupcakes.

Strategy: Build a Table

Cupcakes come in packets of 12.

We can count up from 55 until we get number that is a multiple of 12.

Number of Cupcakes	Divisible by 12?
55	No
56	No
57	No
58	No
59	No
60	Yes

60 is a multiple of 12.

Charlie needs to buy 60 ÷ 12 = **5** packets of cupcakes.

Miss Humphrey's Stickers

Problem: Miss Humphrey bought a packet of stickers to give as prizes to her class.
She gave half of them to Mrs Sutton to use for the class next door.
At the end of the week, she handed out one sticker each to Rhys, Shen and Taylor.
She has twelve stickers left.
How many stickers did Miss Humphrey have before giving any of them away?

Strategy: Work Backwards

First, let's go through the same actions as Miss Humphrey.

Miss Humphrey bought some stickers.	She divided her stickers into 2 equal portions, one set for Mrs Sutton and one set for herself.		Then she gave 3 stickers to her students.		She has 12 stickers left.
? stickers	Divide by 2 →	? stickers	Subtract 3 →	12 stickers	

Now, we can work backwards to find how many stickers Miss Humphrey had in the beginning.

30 stickers	← Multiply by 2	15 stickers	← Add 3	12 stickers
So Miss Humphrey must have bought 30 stickers.	Before she gave half of her stickers to Mrs Sutton, she must have had twice the number of stickers she ended up with.		Before she gave 3 of the stickers to Rhys, Shen and Taylor, she must have had 3 more stickers than she ended up with.	She has 12 stickers left.

Let's check: Miss Humphrey bought 30 stickers.
She gave half of them to Mrs Sutton. After that, she had $30 \div 2 = 15$ stickers.
She gave one sticker each to Rhys, Shen and Taylor. After that, she had $15 - 3 = 12$ stickers.
This matches the question.
Miss Humphrey had **30** stickers before giving any of them away.

Strategy: Work Backwards (Alternative Approach)

Let's draw a bar to represent the packet of stickers.	[Miss Humphrey's stickers]
She gave **half** of them to Mrs Sutton.	[Miss Humphrey | Mrs Sutton]
She gave 3 of them to Rhys, Shen and Taylor.	[Miss Humphrey | 3 | Mrs Sutton]
She had 12 stickers left.	[12 | 3 | Mrs Sutton]
Before giving 3 to Rhys, Shen and Taylor, she must have had $12 + 3 = 15$ stickers.	[15 | Mrs Sutton]
Before giving half to Mrs Sutton, Miss Humphrey had $15 \times 2 = 30$ stickers.	[30]

How Many 2s?

Problem: Sandy writes every whole number from 1 to 100 without skipping any numbers.
How many times will Sandy write the digit “2”?

Strategy: Make an Organised List

Sandy starts from **1** and then writes every whole number up to **100**. So the first time Sandy writes the digit "2" will be for the number 2.	2
Let’s write all of the numbers that will have 2 in the ones place. We can see that there are **10** such numbers. This is because there would be **10** possible digits in the **10**s place if we include **0** (to represent the number **0**2, which we would just write as 2).	2, 12, 22, 32, 42, 52, 62, 72, 82, 92
Now that we have done this, we can use the same idea to write all of the numbers that have the digit 2 in the tens place. Again, there are **10** such numbers.	20, 21, 22, 23, 24, 25, 26, 27, 28, 29
Since the number **22** appears twice - once in each list - we’ll put it aside. There are **9** + **9** = **18** numbers that include a single digit **2**.	2, 12, , 32, 42, 52, 62, 72, 82, 92 20, 21, , 23, 24, 25, 26, 27, 28, 29
The number **22** has two digits “2”. Therefore Sandy would write the digit “2” a total of **18** + **2** = **20** times.	2, 12, , 32, 42, 52, 62, 72, 82, 92 20, 21, , 23, 24, 25, 26, 27, 28, 29 22,

Strategy: Build a Table, and Use Number Sense

Tens \ Ones	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	10	11	12	13	14	15	16	17	18	19
2	20	21	22	23	24	25	26	27	28	29
3	30	31	32	33	34	35	36	37	38	39
4	40	41	42	43	44	45	46	47	48	49
5	50	51	52	53	54	55	56	57	58	59
6	60	61	62	63	64	65	66	67	68	69
7	70	71	72	73	74	75	76	77	78	79
8	80	81	82	83	84	85	86	87	88	89
9	90	91	92	93	94	95	96	97	98	99

Since **100** does not include the digit "2", we can think of Sandy's list as comprising every number from **1** to **99**.

Each number from **1** to **99** has a digit in the ones place.

Each number from **10** to **99** has a digit in the tens place.

We can use a table to show how these numbers are constructed.

We can see that:

- There are **10** "2"s in the ones place, and
- there are **10** "2"s in the tens place.

Therefore Sandy must have written the digit "2" **10** + **10** = **20** times.

123 + 123 + 123 + ...

Problem:

What is the value of the following?

123 + 123 + 123 + 123 + 123 +
123 + 123 + 123 + 123 + 123 +
123 + 123 + 123 + 123 + 123 +
123 + 123 + 123 + 123 + 123

Strategy: Solve a Simpler Related Problem

When we do the times tables, we are adding the same number each time.

For example, to create the two times table, we have:

1 × 2 = 2	2 + 2 = 2 × 2 = 4	2+2+2 = 3 × 2 = 6

and so on. Here, we are adding another **2** each time.

To create the three times table, we have:

1 × 3 = 3	3 + 3 = 2 × 3 = 6	3+3+3 = 3 × 3 = 9

and so on, adding another **3** each time.

For this question, we're adding **123** each time.

1 × 123 = 123	123

123 + 123 = 2 × 123 = 246

123 + 123 + 123 = 3 × 123 = 369

If we do this **20** times, that's the same as working out **20 × 123.**

Method 1:
Written Algorithm

	1	2	3
×		2	0
2	4	6	0

Method 2:
Area Model

	100	20	3
20	2000	400	60

2000 + 400 + 60 = 2460.

The value is
20 × 123 = **2460.**

Strategy: Use Number Sense

Since **123** = 100 + 20 + 3, we can think of

$$\underbrace{123 + 123 + 123 + 123 + 123 + \ldots + 123}_{\text{twenty times}}$$

as being equal to

$$\underbrace{100 + 100 + 100 + 100 + 100 + \ldots + 100}_{\text{twenty times}}$$

$$+ \underbrace{20 + 20 + 20 + 20 + 20 + \ldots + 20}_{\text{twenty times}}$$

$$+ \underbrace{3 + 3 + 3 + 3 + 3 + \ldots + 3}_{\text{twenty times}}$$

Since twenty **100**s = **20** × **100** = **2000**,
twenty **20**s = **20** × **20** = **400**, and
twenty **3**s = **20** × **3** = **60**,
the value of the expression is **2000** + **400** + **60** = **2460**.

Alternative Method

Taking the problem one line at a time:

$\underbrace{123 + 123}$ + $\underbrace{123 + 123}$ + 123

= $\underbrace{246 + 246}$ + 123

= 492 + 123

= 615.

So 123 + 123 + 123 + 123 + 123
+ 123 + 123 + 123 + 123 + 123
+ 123 + 123 + 123 + 123 + 123
+ 123 + 123 + 123 + 123 + 123
= 615 + 615 + 615 + 615
= **2460**.

Problem: There is a three-digit number on my car's number plate.

The hundreds digit is one more than the tens digit.

The ones digit is equal to the hundreds digit multiplied by the tens digit.

No two digits are the same.

What is the three-digit number?

Strategy: Eliminate All But One Possibility

We know that the hundreds digit is one more than the tens digit.

So the number could start with any of the following combinations:

1	0		2	1		3	2	
4	3		5	4		6	5	
7	6		8	7		9	8	

The ones digit is equal to the hundreds digit multiplied by the tens digit.

Clearly, it won't work if the product is a two-digit number.

1	0	0	2	1	2	3	2	6
~~4~~	~~3~~	~~(12)~~	~~5~~	~~4~~	~~(20)~~	~~6~~	~~5~~	~~(30)~~
~~7~~	~~6~~	~~(42)~~	~~8~~	~~7~~	~~(56)~~	~~9~~	~~8~~	~~(72)~~

No two digits are the same.

~~1~~	~~0~~	~~0~~	~~2~~	~~1~~	~~2~~	3	2	6
~~4~~	~~3~~	~~(12)~~	~~5~~	~~4~~	~~(20)~~	~~6~~	~~5~~	~~(30)~~
~~7~~	~~6~~	~~(42)~~	~~8~~	~~7~~	~~(56)~~	~~9~~	~~8~~	~~(72)~~

There's only one possibility remaining.

The three-digit number must be **326**.

268 + 1375 + 6179 – 168 – 1275 – 6079

Problem: What is the value of the following?

268 + 1375 + 6179 – 168 – 1275 – 6079

Strategy: Convert to a More Convenient Form

To make this problem easier to think about, we can pretend that we are counting items - for example, beans.

To begin with, we have the following:	268	+	1375	+	6179	The next thing we need to do is take away 168 beans. If we take the 168 beans out of the bag that currently has 268 beans, then the subtraction is much simpler.
We now have the following:	268 – 168 = 100	+	1375	+	6179	and we need to take away 1275 beans. This time, it might be simpler if we take the 1275 beans out of the bag that currently has 1375 beans.
Finally, we have:	268 – 168 = 100	+	1375 – 1275 = 100	+	6179	and we need to take away 6079 beans. We can take the 6079 beans out of the bag that currently has 6179 beans.
We are left with this many beans.	268 – 168 = 100	+	1375 – 1275 = 100	+	6179 – 6079 = 100	Each bag now holds 100 beans. Therefore, the result is 100 + 100 + 100 = **300**.

Strategy: Split Strategy

We can split all of the numbers into their place value components, resulting in the following:

268 + 1375 + 6179 – 168 – 1275 – 6079

= 200 + 60 + 8 + 1000 + 300 + 70 + 5 + 6000 + 100 + 70 + 9 – 100 – 60 – 8 – 1000 – 200 – 70 – 5 – 6000 – 70 – 9

Grouping by place value, we have:

$$= \underbrace{1000 + 6000 - 1000 - 6000}_{0} + \underbrace{200 + 300 + 100 - 100 - 200}_{300} + \underbrace{60 + 70 + 70 - 60 - 70 - 70}_{0} + \underbrace{8 + 5 + 9 - 8 - 5 - 9}_{0}$$

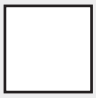
= 0 + 300 + 0 + 0

Therefore the value of the expression is 0 + 300 + 0 + 0 = **300**.

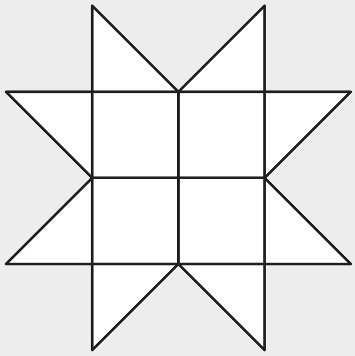
Problem:

I am using square tiles to make a pattern.

The tiles can be cut to make different shapes.

All of the tiles are this size: 

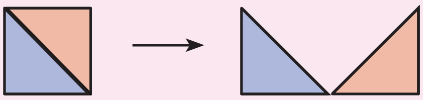
What is the smallest number of tiles I will need to make this 8-pointed star?



Strategy: Divide a Complex Shape

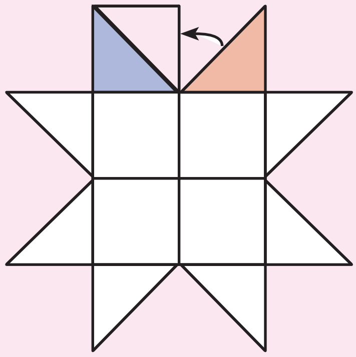
A square can be cut diagonally to make two of the triangular shapes.

Both of the resulting triangles are exactly the same.



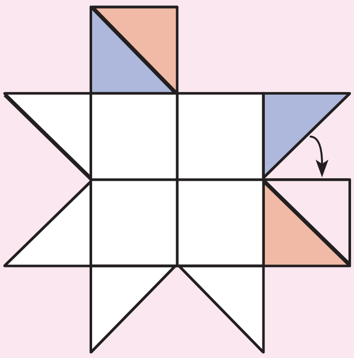
If I put two triangles together, they would form a square.

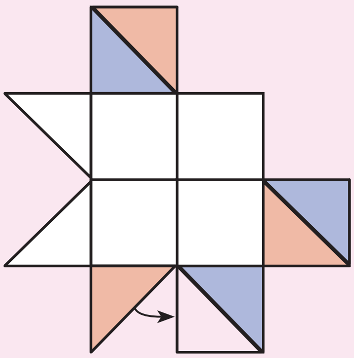
This means that I can make the two triangle tiles on the top of the star, using just one square tile.

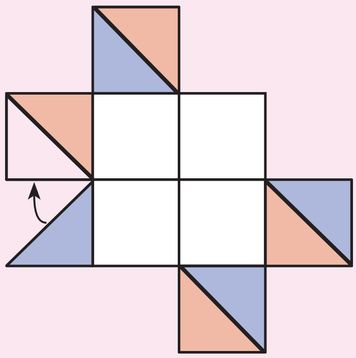


Other pairs of triangles can also be created by cutting a single square tile.

We can see that it works because, in each case, we can put two triangles together to form a square.

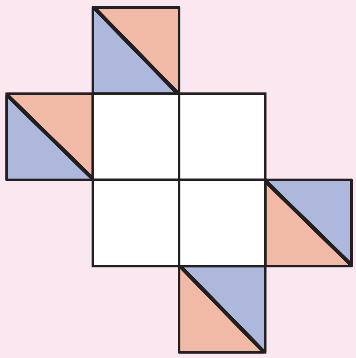






There are enough tiles here to make the entire shape.

The smallest number of tiles required to make the star shape is **8**.



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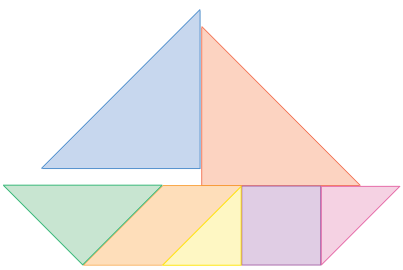

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