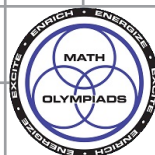


Maths Olympiad Contest Problems

Volume 4

Exploring Maths Through Problem Solving

**Contains Maths Olympiad Papers
From Australia 2014 to 2017
and USA 2013/14 to 2016/17**



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Contest Problem Types

Many but not all contest problems can be categorised. This is useful if you choose to work with several related problems even if they involve different concepts.

KEY: problems are organised by type and are coded by **page number and problem placement** on that page. For example, “**Factors: 29B, 154AE**” refers to three questions, each involving factors: problem B on page 29, and problems A and E on page 154.

Note: Contest problems from the different divisions are arranged as follows:

Australian Division J:	pages	2 - 21
USA Division E:	pages	23 - 42
Australian Division S:	pages	148 - 167
USA Division M:	pages	169 - 188

Each *FOLLOW-UP* problem is located after a model solution to a contest problem, is related conceptually to it, and usually extends or expands an aspect of it.

A

Addition patterns — see **Patterns**

Age problems: 14B, 18D, 21D, 38E, 179D

Algebraic thinking: 3C, 7C, 8D, 10C, 12DE, 13A, 15AB, 18D, 19D, 21D, 28E, 31A, 38E, 41C, 42B, 152E, 153C, 156B, 158D, 160D, 161B, 163D, 165A, 167A, 169D, 173D, 175B, 181B, 182E, 185D, 186A

— Also see *Digit problems*; *Coin problems*; *Age problems*

Alphanumeric problems — see *Cryptarithms*

Angles: 164C, 166E, 177E, 188D

Area: 3D, 10E, 17D, 21C, 23D, 26E, 27C, 28D, 30C, 32E, 35A, 36C, 40D, 41E, 156D, 158C, 159B, 160B, 163C, 167D, 173C, 175D, 181E, 186D

— **and perimeter:** 5BE, 13C, 14C, 20D, 26C, 170B

— Also see *Circles and area*; *Painted cube problems*

Arithmetic operations and properties: 3A, 4A, 6A, 7A, 8A, 9A, 10B, 11A, 12A, 13B, 14A, 17B, 18A, 27A, 28A, 29A, 30A, 32AB, 33A, 36AB, 37E, 38A, 39AB, 40A, 42A, 148A, 156A, 159A, 179A, 182A, 184A, 185A

Arithmetic sequences and series — see *Sequences and Series*; *Patterns*

Averages (arithmetic means) — see *Mean*

C

Calendars: 33B, 38B, 40C, 174A, 179D

— Also see *Cycling numbers*; *Remainders*

Certainty problems: 17C, 180E

Circles:

— and area: 156D, 182D

— and circumference: 177E

Clock problems: 20B, 36D, 152D, 160E, 176B

Coin problems: 12E, 31A, 32C

Combinations and permutations: 24B, 151B, 162C, 164D, 169E, 172D, 179E

Common multiples — see *Multiples*

Congruent figures: 159E

Consecutive numbers: 3A, 6B, 14A, 17B, 25D, 28A, 40B, 165B, 185D

— **Consecutive odd or even numbers:** 24B, 148B, 158B

Coordinates — see *Graphs*

Cryptarithms: 2D, 6E, 9DE, 13DE, 19E, 23E, 25CE, 33C, 42D, 153C, 157E, 158E, 165E, 167C

— Also see *Digit problems*

Cubes and rectangular solids: 26E, 40D, 41E, 149B, 153A, 158C, 173C

— **Painted cube problems:** 4E, 6D, 18E, 29D, 31E, 35E, 42E, 148D, 156C, 172E

Cubic numbers — see *Square and cube numbers*

Cycling numbers: 11C, 154C, 159D, 166B, 174A

D

Decimals — see *Fractions*

Digit problems: 149A, 176A, 183C

— Also see *Cryptarithms*; *Divisibility*

Distance problems — see *Motion problems*

Distributive property: 3B, 17A, 26A, 31B, 34B, 150A, 177A

Divisibility: 4B, 14D, 16A, 20C, 23C, 26D, 30E, 31C, 34A, 36E, 42C, 157B

— Also see *Factors*; *Multiples*; *Cycling numbers*

Draw a diagram: 25B, 151C, 153B, 154B, 174C

— Also see *Graphs*

E

Estimation: 164A, 166C

Even vs. odd numbers — see *Parity*

Exponents: 151A, 153D, 155C, 157DE, 158A, 159C, 165D, 167B, 170D, 177C, 178D, 180A, 185B

JUNIOR OLYMPIAD PROBLEMS

**Division J (Australian)
Sets 1-4**

**Division E (USA)
Sets 5-8**

Set 1: Olympiad 2

2A

3

Minutes

79%

Find the sum: $137 + 138 + 139 + 140 + 141 + 142 + 143$.

2B

4

Minutes

65%

What is the value of $(1 \times 50) + (22 \times 50) + (28 \times 50) + (49 \times 50)$?

2C

5

Minutes

40%

Sarah and Connor went bowling together. Sarah rented bowling shoes and played three games, for a total cost of \$9.50. Connor rented bowling shoes and played five games, for a total cost of \$14.50.

How much does it cost to rent bowling shoes?

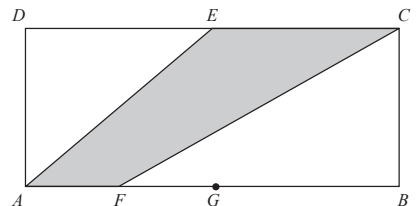
2D

6

Minutes

27%

$ABCD$ is a rectangle with an area of 80 square metres. Point E is the midpoint of DC , Point G is the midpoint of AB , and Point F is the midpoint of AG .



What is the area in square metres of the shaded trapezium $AFCE$?

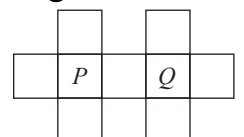
2E

7

Minutes

44%

Each digit from 1 to 9 is placed, one per box, in the diagram shown. The five numbers in the middle row add to 18. The first column of three adds to 16, and the last column of three adds to 22.



What is the value of $P + Q$?

SENIOR OLYMPIAD PROBLEMS

**Division S (Australian)
Sets 9-12**

**Division M (USA)
Sets 13-16**

Set 9: Olympiad 1

1A

3

Minutes

75%

Find the difference when $12 \div 6 \times 2$ is subtracted from $12 \times 6 \div 2$.

1B

4

Minutes

34%

Find the difference when the sum of the first ten even positive integers is subtracted from the sum of the first ten positive integer multiples of 3.

1C

4

Minutes

28%

One million seconds is closest to N days.

Find the whole number N .

[Use 1 day = 24 hours]

1D

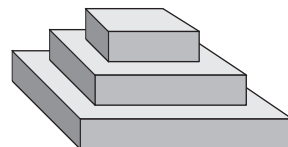
5

Minutes

25%

A tower is formed by three rectangular prisms stacked on top of one another as shown. Each prism is 1 metre high.

The prism on the bottom has a base that is 7 metres \times 6 metres, the middle prism has a base that is 5 metres \times 4 metres, and the top prism has a base that is 3 metres \times 2 metres.



The tower, excluding the bottom of the largest prism, is to be painted. Find the area to be painted, in square metres.

1E

6

Minutes

13%

In the partially completed square array shown, each row, column, and diagonal has the same sum.

Find x .

20	15	x
18		

Hints: Division S**Set 12: Olympiad 1**

- 1A. The answer doesn't have to be a fraction.
- 1B. Replace each bracketed term with the equivalent fraction.
- 1C. Try adding the line DB .
- 1D. What might y be if $xy^2 = 12$?
- 1E. How far right has the pattern travelled before beginning the 10th horizontal step?

Set 12: Olympiad 2

- 2A. What value must N^2 be close to?
- 2B. Suppose the average of four numbers is 3. What would happen to the average if a fifth number, "3", was then added to the set?
- 2C. Try dividing the shape into triangles.
- 2D. List the possible 2-digit numbers in an organised way.
- 2E. Start by considering all of the numbers that satisfy (3). Then consider if there are any numbers that satisfy (1), (2) and (4) but not (3).

Set 12: Olympiad 3

- 3A. Try guessing a number of pencils for Cheryl.
- 3B. Can the numbers be rearranged to make the calculation more convenient?
- 3C. Is there a combination of side lengths that will not make a triangle?
- 3D. Write 2017^8 in expanded form. What expression would be equivalent to $\sqrt{2017^8}$?
- 3E. For which letter is it most important to have a small numeric value?

Set 12: Olympiad 4

- 4A. How far does the car travel in one minute?
- 4B. Is there a pattern?
- 4C. What values must this product be between?
- 4D. Try making an organised list of the numbers less than 200 whose digits sum to 8.
- 4E. Try splitting each exterior angle along the grid lines. Can the pieces be combined in a more convenient way?

Set 12: Olympiad 5

- 5A. What must be the price of the third widget?
- 5B. Rewrite each term in the alternative form.
- 5C. What must be the value of U ?
- 5D. You could draw a diagram.
- 5E. What sums might result if the cards with "7" on them were never chosen?