Maths Olympiad Contest Problems

For Primary and Middle Schools

Australian Edition

Exploring Maths Through Problem Solving

Contains APSMO Maths Olympiad Papers
Prior to 1996

Contents

Preface to Australian Edition

The Australasian Problem Solving Mathematical Olympiads (APSMO) Inc has been offering Mathematical Olympiads based on Dr Lenchner's model to schools throughout Australia, New Zealand and surrounding countries since 1987. The annual inter-school Olympiads are held five times a year between May and September.

We take this opportunity to thank Dr Lenchner for his permission to reprint his excellent text with modifications specific to Australian education.

This text is identical to Dr Lenchner's original text with the following modifications:

- Australian spelling
- Changes in nomenclature such as imperial to decimal measurements, American coinage to Australian coinage
- All Olympiad questions remain true to the original. In certain situations the answers may differ to the original answers, however all care has been taken to ensure that the purpose and solution methods remain unchanged. Consequently, we have continued to use 1c and 2c coins although they are no longer in use in Australia.
- Where it was not possible to change a question without altering the solution methods or intention of the question, a note has been included within the question text as clarification for students. For example: [Note: There are 3 feet in 1 yard].

Thank you to Dr Anne Prescott, lecturer in primary and secondary mathematics education at the University of Technology, Sydney, for her valuable assistance in reviewing the alterations and ensuring that the modified questions contained within this text are correct and suitable for Australian students.

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Australasian Problem Solving Mathematical Olympiads (APSMO) Inc

Introduction

The Problem Solving Process

Understanding the Problem

Before you try to solve a problem, make sure that you understand the wording of the problem, its question, and any special words it might contain such as factor, digit, diagonal, and so forth. Does the problem give you too little, just enough, or too much information? Can you restate the problem in your own words? Can you guess what the answer will look like?

Planning to solve the problem

You need to have a plan of action to solve a problem. Such plans are called *strategies*.

The following are some strategies that are used more frequently than others.

Find a Pattern
Draw a Picture or Diagram
Make an Organised List
Make a Table
Work Backwards
Use Reasoning

Carrying Out the Plan

You will observe that each problem in this book has a suggested time limit which begins after you have read the problem and are ready to begin. Select a strategy from the above list or use one of your own choice. You may also want to use a combination of strategies. Now try to solve the problem. If you are not able to do the problem within the recommended time limit, ignore the time limit and continue to work on the problem until you have a solution.

Speed is not important!

As you become more experienced with different strategies, mathematical ideas and principles, and develop your skills, the amount of time you need to do a problem will decrease naturally. If you have difficulty with a computation, get help from your teacher, a parent, or another student. If your strategy doesn't seem to be working, try a different strategy. If you are still "stumped", go on to another problem. Later, you may want to again try to do the problem that stumped you. Perhaps you will think of a way of doing the problem in the time that has elapsed. If that doesn't help, read *Using Different Parts of the Book* which begins on page 15.

Olympiad 1

1.

4 min.

Suppose today is Tuesday. What day of the week will it be 100 days from now?

2.

5 min.

I have four 3c-stamps and three 5c-stamps. Using one or more of these stamps, how many different amounts of postage can I make?

3.

5 min.

Find the sum of the counting numbers from 1 to 25 inclusive.

In other words, if $S = 1 + 2 + 3 + \cdots + 24 + 25$, find the value of S.

4.

6 min.

In a stationery store, pencils have one price and pens have another price. Two pencils and three pens cost 78c. But three pencils and two pens cost 72c. How much does one pencil cost?

5.

5 min.

A work crew of 3 people requires 3 weeks and 2 days to do a certain job. How long would it take a work crew of 4 people to do the same job if each person of both crews works at the same rate as each of the others?
[Note: each week contains six work days.]

Olympiad 1

1) What day will it be 7 days from now? 14 days from now? 77 days from now?
2) Make an organised list of the different amounts starting with the 3c-stamps.
3) Rewrite the series in reverse order placing each term directly under the term of the given series. Examine each vertical pair of terms.
4) How much will 5 pens and 5 pencils cost?
5) How long would it take one person to do the entire job alone?

Olympiad 2

1) Act it out.
2) Try using half of the coins as 5c and the other half as 20c.
3) How many square centimetres are there in the rectangular sheet 24 cm by 36 cm?
4) If the average score for 4 games is 145, what is the total score for the 4 games?
5) If you counted from 1 on, how frequently would "1" appear in the units place? tens place? hundreds place?

Olympiad 3

1) Try a simpler problem with 2, 3, or 4 children.
2) Average speed is the total distance divided by the total time.
3) If the sum of the digits of a number is divisible by 9, the number is also divisible by 9.
4) $\frac{1}{9\times 10}=\frac{1}{9}-\frac{1}{10}$
5) Compare the terms of the sequence with multiples of 3, starting with 3.

Olympiad 4

1) Make 1.75 and 1¼ either both decimals or both mixed numbers.
2) $A\times AB = 114$. Try different values for A starting with 2.
3) Experiment with 2 lines and count the sections. Then try 3 lines, and then 4 lines.
4) Could A be less than 3?
5) Do (6*8) first.

Olympiad 5

1) What is the average of the five numbers?
2) How many times larger than 600 square feet is 600 square yards?
3) Work from the bottom up.
4) What is the largest number that the two-digit numbers can divide exactly?
5) Try packaging some marbles in the larger boxes and examine what is left over.

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When a page reference is given and followed by a number in parentheses, the latter denotes a section number on the page. For example, 248(2) represents page 248, section 2 (or problem 2). For definitions of terms see the Glossary. When a topic is listed below and shown in italics, it appears in Problem Types where references are made to Olympiad contest numbers and problem numbers (see pages 271 and 272).